



QUESTION 2. A population of butterflies lives on a meadow, surrounded by forest. We want to investigate the dynamics of the population. We denote the number of butterflies at the beginning of season  $t$  by  $x_t$ . Over the course of a season, 30% of the butterflies that were there at the beginning die. During each season, 20 new butterflies arrive from other meadows.

a) Write the DTDS for the number of butterflies, i.e.  $x_{t+1} = \boxed{0.7x_t + 20}$

b) The updating function is  $f(x) = \boxed{0.7x + 20}$

c) Starting with 40 butterflies in season 0, calculate their number in seasons 1, 2, 3.

$x_1 = \boxed{48}$        $x_2 = \boxed{53.6}$        $x_3 = \boxed{57.52}$

d) The fixed point of the DTDS is  $x^* = \boxed{\frac{20}{1-0.7} = 66.67}$

e) The solution of the DTDS in terms of a general initial condition  $x_0$  is

$x_t = \boxed{(0.7)^t (x_0 - x^*) + x^*}$

f) In the space below, draw the cobweb for this DTDS, starting at  $x_0 = 40$ . Also draw the solution as a function of time.

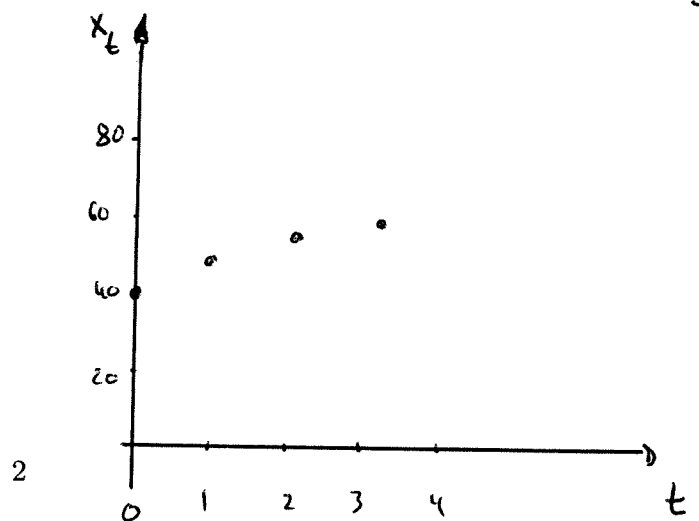
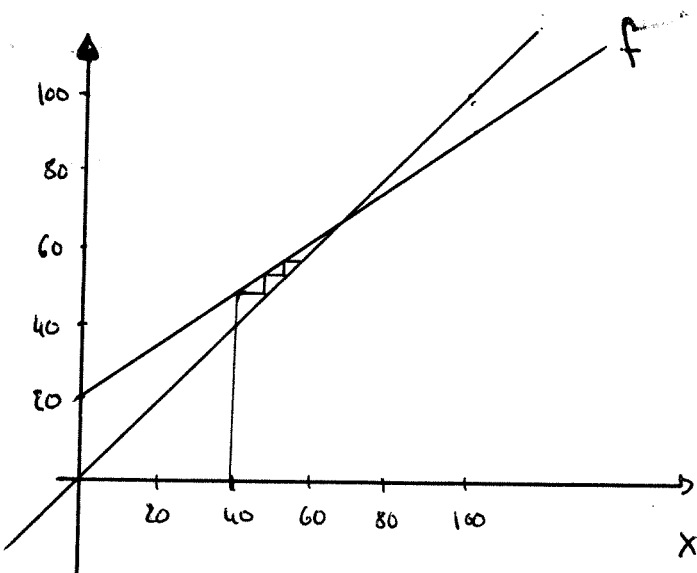
g) Suppose that through some conservation measures, we can improve the quality of the pond and reduce the death rate of the butterflies. To which level do we have to reduce the death rate if we want the steady state butterfly population to be 100? Answer: The death rate should be

$\boxed{20}\%$ . Let  $d$  be the death rate.

Draw your cobweb and solution here. The DTDS is  $x_{t+1} = (1-d)x_t + 20$

The steady state is  $x^* = \frac{20}{1-(1-d)} = \frac{20}{d}$

so  $d = \frac{1}{5}$



QUESTION 3. Consider the nonlinear DTDS  $x_{t+1} = \frac{rx_t}{1+0.2x_t}$ , where  $r > 0$  is some parameter.

a) The fixed point(s) of this DTDS is (are)  $x=0$  and  $x=5(r-1)$

(Note: the answer may contain parameter  $r$ .)

b) Now set  $r = 3$ . Calculate  $x_1, x_2, x_3$  starting form  $x_0 = 8$ .

$x_1 = 9.23$        $x_2 = 9.73$        $x_3 = 9.91$

c) Keeping  $r = 3$ , calculate  $x_1, x_2, x_3$  starting form  $x_0 = 12$ .

$x_1 = 10.59$        $x_2 = 10.19$        $x_3 = 10.06$

d) Do you observe a trend in these values?

When  $x_0 < 10$ , the values increase, but stay below 10

When  $x_0 > 10$ , the values decrease, but stay above 10

The positive fixed point is  $x = 10 = 5(3-1)$

Show your work here.

$$x = \frac{rx}{1+0.2x} \text{ has solution } x=0 \text{ and } 1+0.2x = r \text{ or } x = 5(r-1)$$