MAT 2379, Introduction to biostatistics

Solution to Assignment 1

Due date: Friday September 25, 2015

$Total = 100$ marks

Problem 2.4 (10 marks) (a) Both parents have the genotype Ff.

Female Gamete	Male Gamete $\frac{1}{2}F$	$\frac{1}{2}f$
$\frac{1}{2}F$	$\frac{1}{4}FF$ (frizzled)	$\frac{1}{4}Ff$ (slightly frizzled)
$\frac{1}{2}f$	$\frac{1}{4}Ff$ (slightly frizzled)	$\frac{1}{4}ff$ (normal)

The offspring can be frizzled with probability 1/4, normal with probability 1/4, and slightly frizzled with probability 1/2.

(b) Male $= Ff$; Female $= ff$.

Female Gamete	Male Gamete $\frac{1}{2}F$	$\frac{1}{2}f$
ff	$\frac{1}{2}Ff$ (slightly frizzled)	$\frac{1}{2}ff$ (normal)

The offspring can be slightly frizzled with probability 1/2, and normal with probability 1/2.

(c) Male $= Ff$; Female $= FF$.

Female Gamete	Male Gamete $\frac{1}{2}F$	$\frac{1}{2}f$
FF	$\frac{1}{2}FF$ (frizzled)	$\frac{1}{2}Ff$ (slightly frizzled)

The offspring can be slightly frizzled with probability 1/2, and frizzled with probability 1/2.

Problem 4.5 (15 marks) Let D be the event that the individual dies and that S be the that the individual has sickle cell allele.

(a) We know that $P(S) = 0.2$, $P(D|S') = 0.15$, $P(D|S) = 0.025$. Hence, by the total probability rule

$$\begin{aligned} P(D) &= P(D|S)P(S) + P(S|D')P(S') \\ &= (0.025)(0.2) + (0.15)(0.8) = 0.125. \end{aligned}$$

(b)

$$P(S|D) = \frac{P(S \cap D)}{P(D)} = \frac{P(D|S)P(S)}{P(D)} = \frac{(0.025)(0.2)}{0.125} = 0.4.$$

Problem 4.7 (10 marks) Let T be the event that the individual has tuberculosis and H be the event that the individual has HIV. We know that $P(T) = 0.00045$ and $P(H|T) = 0.08$. We want

$$P(T \cap H) = P(H|T)P(T) = (0.08)(0.00045) = 0.000036.$$

Problem 4.8 (15 marks) We know that $P(H|T') = 0.0016$.

(a) By the total probability rule

$$\begin{aligned} P(H) &= P(H|T)P(T) + P(H|T')P(T') \\ &= (0.08)(0.00045) + (0.0016)(1 - 0.00045) \\ &= 0.00164 \ \ (\text{ or } 0.164\%) \end{aligned}$$

(b)

$$P(T|H) = \frac{P(H|T)P(T)}{P(H)} = \frac{(0.08)(0.00045)}{0.00164} = 0.022.$$

Problem 8.2 (15 marks) (a) There are 4 possible cases: (1) $I^AI^A \times I^BI^B$; (2) $I^AI^A \times I^Bi$; (3) $I^Ai \times I^BI^B$; (4) $I^Ai \times I^Bi$.

(b) Case (1): $I^AI^A \times I^BI^B$

Female Gametes	Male Gametes I^B
I^A	I^AI^B (type AB)

Case (2): $I^AI^A \times I^Bi$

Female Gametes	Male Gametes $\frac{1}{2}I^B$	$\frac{1}{2}i$
I^A	$\frac{1}{2}I^AI^B$ (type AB)	$\frac{1}{2}I^Ai$ (type A)

Case (3): $I^Ai \times I^BI^B$

Female Gametes	Male Gametes I^B
$\frac{1}{2}I^A$	$\frac{1}{2}I^AI^B$ (type AB)
$\frac{1}{2}i$	$\frac{1}{2}iI^B$ (type B)

Case (4): $I^Ai \times I^Bi$

Female Gametes	Male Gametes $\frac{1}{2}I^B$	$\frac{1}{2}i$
$\frac{1}{2}I^A$	$\frac{1}{4}I^AI^B$ (type AB)	$\frac{1}{4}I^Ai$ (type A)
$\frac{1}{2}i$	$\frac{1}{4}iI^B$ (type B)	$\frac{1}{4}ii$ (type O)

(c) Yes, it is possible that their offspring has type O blood. This happens in Case (4).

Problem 8.3 (10 marks) Let A be the event that the patient suffers from memory loss, and B be the event that the patient has difficulty completing familiar tasks. We know that $P(A) = 0.85, P(B) = 0.78$ and $P(A \cap B) = 0.67$.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.85 + 0.78 - 0.67 = 0.96$.

(b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.96 = 0.04$.

Problem 8.13 (10 marks) Let A be the event that the fish has a tumor, and B the event that the fish is young. We know that $P(A) = 0.23$, $P(B) = 0.05$, and $P(A \cap B) = 0.005$.
(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.23 + 0.05 - 0.005 = 0.275$.
(b) $P(B \cap A') = P(B) - P(A \cap B) = 0.05 - 0.005 = 0.045$.
(c) $P(A \cap B') = P(A) - P(A \cap B) = 0.23 - 0.005 = 0.225$.

Problem 8.14 (15 marks) Let A be the event that the donor is HIV positive and that B is the event that the donor is positive for herpes. We know that $P(A) = 0.01$, $P(B) = 0.02$, and $P(A' \cap B) + P(A \cap B') = 0.015$. We first calculate $P(A \cap B)$. Note that

$$P(A) + P(B) = P(A' \cap B) + P(A \cap B') + 2\,P(A \cap B).$$

Hence, $P(A \cap B) = (0.03 - 0.015)/2 = 0.0075$. Next, we calculate

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.01 + 0.02 - 0.0075 = 0.0225.$$

The desired probability is

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.0225 = 0.9775.$$