gent.

(a) True

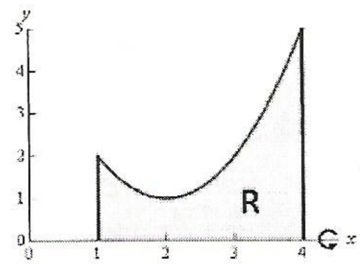
(b) False

(1 *mark*) 2. $\displaystyle\int_1^\infty \frac{dx}{x^{\sqrt{2}}}$ is convergent.

(a) True

(b) **False**

(1 *mark*) 3. It would be easiest to use the method of cylindrical shells to calculate the volume of the solid formed when the region R is rotated about the x-axis.



(a) **True**

(b) **False**

(1 *mark*) 4. Simpson's Rule uses ____________ to approximate a curve.

(a) straight line segments

(b) parabolas

(c) cubic equations

(d) quartic equations

(2 *marks*) 5. Convert $(-2, 2)$ (given in Cartesian coordinates) into polar coordinates.

(a) $\left(2, \frac{3\pi}{4}\right)$ X

(b) $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ ✓

(c) $\left(-2\sqrt{2}, \frac{3\pi}{4}\right)$

(d) $\left(-2\sqrt{2}, \frac{7\pi}{4}\right)$

$-2 = x = r\cos\theta$ $\quad r = \frac{-2}{\cos\theta}$

$\frac{-\pi}{4}$

$r = -2.82$ $\quad 2 = y = r\sin\theta$ $\quad r = \frac{2}{\sin\theta}$

$\theta = -\frac{\pi}{4}$ $\quad r = -2.82$

$\frac{-2}{\cos\theta} = \frac{2}{\sin\theta}$

$\frac{-\pi}{4}$

$\tan\theta = -1$

(2 *marks*) 6. Calculate $\dfrac{d^2y}{dx^2}$ for the parametric curve defined by the equations $x = \ln t$, $y = t^2$.

(a) $-2t^2$

(b) $4t^2$

(c) $2t^2$

(d) $4t$

$x = \ln t$ $\quad y = t^2$

$\frac{dx}{dt} = \frac{1}{t}$ $\quad \frac{dy}{dt} = 2t$

$\frac{2t}{1/t} = 2t^2$

$\frac{4t}{\frac{1}{t}}$

Over

(8 *marks*) 7. By using a suitable trigonometric substitution, evaluate the integral $\int \frac{dx}{x\sqrt{x^2+4}}$.

$\int \frac{1}{x\sqrt{x^2+4}}\,dx$ let $x = 2\tan\theta$
$dx = 2\sec^2\theta\, d\theta$

$= \int \frac{1}{(2\tan\theta)\sqrt{4\tan^2\theta+4}}\,(2\sec^2\theta\, d\theta)$

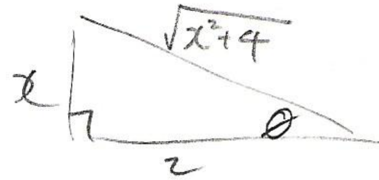
$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$= \int \frac{2\sec^2\theta}{2\tan\theta\sqrt{4(\tan^2\theta+1)}}\,d\theta$

$x = 2\tan\theta$

$= \int \frac{\sec^2\theta}{\tan\theta\sqrt{4\sec^2\theta}}\,d\theta$



$\tan\theta = \frac{x}{2}$

$= \int \frac{\sec^2\theta}{\tan\theta(2\sec\theta)}\,d\theta$

$= \frac{1}{2}\int \frac{\sec\theta}{\tan\theta}\,d\theta$

$\cot = \frac{1}{\tan} = \frac{2}{x}$

$\csc = \frac{1}{\sin} = \frac{\sqrt{x^2+4}}{x}$

$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

$= \frac{1}{2}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\,d\theta$

8

$= \frac{1}{2}\int \frac{1}{\sin\theta}\,d\theta$

$= \frac{1}{2}\int \csc\theta\, d\theta$

$= \frac{1}{2}(-\ln|\cot\theta + \csc\theta|) + C$

$\boxed{= \frac{1}{2}\left(-\ln\left|\frac{2}{x} + \frac{\sqrt{x^2+4}}{x}\right|\right) + C}$

(5 *marks*) 8. Use Simpson's rule with $n = 4$ to approximate $\int_0^1 \cos(x^2)\,dx$.

$\Delta n = \frac{1-0}{4}$
$= \frac{1}{4}$

$S_4 = \frac{\Delta n}{3}\left[f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + f(1)\right]$

4

$f($

$0 \quad 0.25 \quad 0.5 \quad 0.75 \quad 1$

$= \frac{1}{12}\left[\cos(0^2) + 4\cos(0.25^2) + 2\cos(0.5^2) + 4\cos(0.75^2) + \cos(1^2)\right]$

$= \frac{1}{12}\left[17.50137\right]$

degree mode?

(6 *marks*) 9. Evaluate the improper integral $\int_0^2 \frac{e^x}{e^x - 1}\,dx$ or show that it is divergent.

$f(x) = \frac{e^x}{e^x - 1}$ $f(2) = \frac{e^2}{e^2-1}$ $f(0) = \frac{1}{1-1} = \frac{1}{0} = $ DIVE

$\lim_{t\to 0^+} \int_t^2 \frac{e^x}{e^x - 1}$ let $u = e^x - 1$, $du = e^x dx$

$= \lim_{t\to 0^+} \int_{x=t}^{x=2} \frac{1}{u}\,du$

$= \lim_{t\to 0^+} \ln|u| \Big]_{x=t}^{x=2}$

6

$= \lim_{t\to 0^+} \ln|e^x - 1| \Big]_t^2$

$= \lim_{t\to 0^+} \left(\ln|e^2-1| - \underbrace{\ln|e^t-1|}_{\to -\infty} \right)$

$= \ln|e^2 - 1| - (-\infty)$

$= \infty$

$\boxed{\therefore \text{ it is divergent}}$

(6 *marks*) 10. Use the Comparison Theorem to determine whether the integral $\int_1^\infty \frac{|\sin(x)|}{\sqrt{x^3+1}}\,dx$ is convergent or divergent.

Note: $1 \geq |\sin(x)| \geq 0$ $x^3+1 \geq x^2+1$

ok!

$\frac{3x^2}{\sqrt{x^3+1}} \geq \frac{1}{\sqrt{x^3+1}} \geq \frac{|\sin(x)|}{\sqrt{x^3+1}} \geq \frac{0}{\sqrt{x^3+1}} = 0$ $\frac{1}{\sqrt{x^3+1}} \leq \frac{1}{\sqrt{x^2+1}}$

$\frac{1}{x^3+1}$ $3x^2$

$\frac{1}{x} \geq \frac{1}{x^2} \geq \frac{1}{x^3} \geq \frac{1}{x^4}$ show convergence ☆ $3x^2 \geq |\sin(x)|$ for $x > 1$

1 $\frac{1}{x^p}$ $p<1$ poor form $1 \geq |\sin x| > 0$

$= 2\sqrt{x^3+1}\Big]_1^\infty$

$= \infty$ ← doesn't say anything about given integral

$\frac{x^3+1}{x^3+1}$

use an integral

$\int^\infty \frac{1}{x}\,dx = $ divergent $\int^\infty |\sin(x)|\,dx$

$]^\infty$

(4 *marks*) 11. Set up but **do not evaluate** an integral to calculate the area of the surface obtained by rotating the curve $y = 4 - x^2$, $0 \le x \le 2$ about the y–axis.

$y = 4 - x^2$ $\quad x = \pm\sqrt{4-y}$

$y - 4 = -x^2$

$4 - y = x^2$

$c = 4 - 0^2 = 4$

$d = 4 - 2^2 = 0$

1

$\int_c^d \pi\, g(y)^2\, dy$

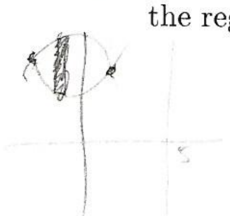
$= \int_4^0 \pi (4-y)\, dy$

$\int_0^2 2\pi x \sqrt{1 + (-2x)}\, dx$

$= \int_0^2 2\pi x \sqrt{1-2x}\, dx$

$\cancel{= 2\pi \left[\int_0 \sqrt{x} - \sqrt{2} \int^2 x\, dx \right.}$ -

(4 *marks*) 12. Set up but **do not evaluate** an integral to calculate the volume of the solid obtained by rotating the region bounded by $y = -x^2 + 5x$ and $y = x^2 - 5x + 8$ about the line $x = 5$.



4

$\int_1^4 2\pi (5-x)\left[(-x^2+5x) - (x^2-5x+8)\right] dx$

$= 2\pi \int_1^4 (5-x)(-2x^2+10x-8)\, dx$

$-x^2 + 5x = x^2 - 5x + 8$

$2x^2 - 10x + 8 = 0$

$x^2 - 5x + 4 = 0$

$x = 1, 4$

(7 *marks*) 13. Find an equation, **solved explicitly for** y, that satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{\sin^3 x \cos x}{2y}$ and passes through the point $\left(\frac{\pi}{2}, -1\right)$.

$\frac{dy}{dx} = \frac{\sin^3 x \cos x}{2y}$

$\int 2y\, dy = \int \sin^3 x \cos x\, dx$

$y^2 = \frac{1}{4}\sin^4 x + C$

$y = \pm\sqrt{\left(\frac{1}{4}\sin^4 x + C\right)}$

$y = -\sqrt{\frac{\sin^4 x}{4} + \frac{3}{4}}$

2

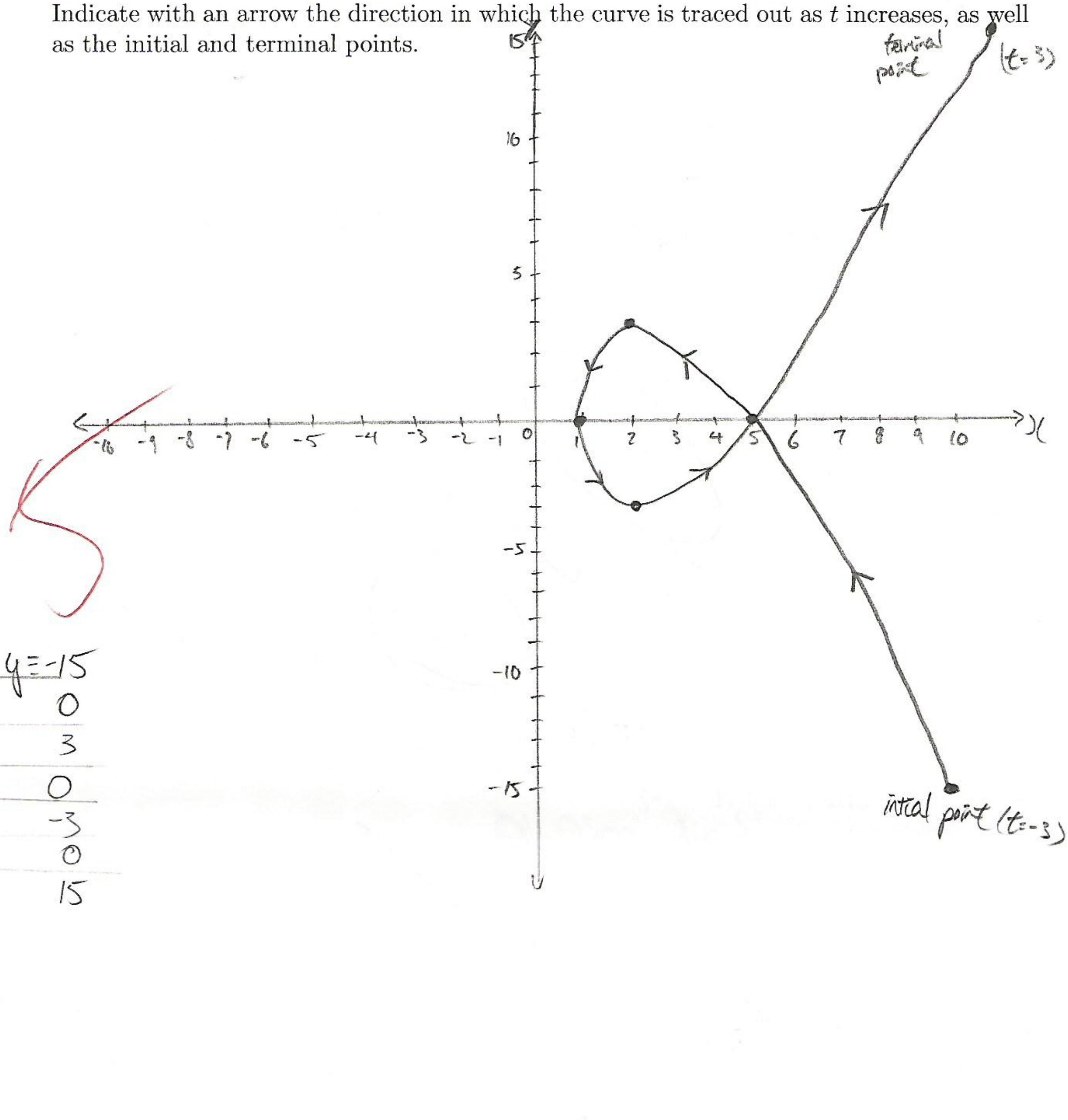
$-1 = -\sqrt{\frac{1}{4}\sin^4\left(\frac{\pi}{2}\right) + C}$

$-1 = -\sqrt{\frac{1}{4}(1)^4 + C}$

(8 *marks*) 14. (a) Sketch the parametric curve

$$x = t^2 + 1 \qquad y = t^3 - 4t \qquad -3 \le t \le 3.$$

Indicate with an arrow the direction in which the curve is traced out as t increases, as well as the initial and terminal points.



$t=-3$	$x=10$	$y=-15$
-2	5	0
-1	2	3
0	1	0
1	2	-3
2	5	0
3	10	15

(b) Set up but **do not evaluate** an integral to calculate the length of the curve.

$x = t^2 + 1 \qquad y = t^3 - 4t$

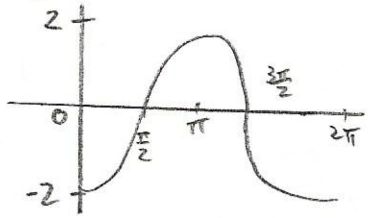
$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2 - 4$

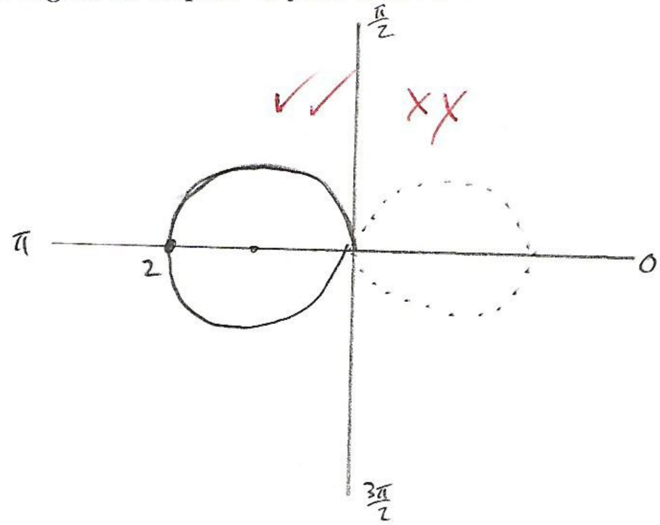
$$L = \int^{3} \sqrt{(2t)^2 + (3t^2-4)^2}\, dt$$

(9 *marks*) 15. Find the area of the region R that lies inside the curve $r = -2\cos\theta$ but outside the curve $r = 1$. Include a sketch of the curves and the region R as part of your solution.

$r = -2\cos\theta = f(\theta)$

$y = -2\cos x$





$$\text{Area} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}[f(\theta)]^2 \, d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}[-2\cos\theta]^2 \, d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}(4\cos^2\theta) \, d\theta$$

$$= 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2\theta \, d\theta$$

$$= 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}(1+\cos 2\theta) \, d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1+\cos 2\theta \, d\theta$$

$$= \theta + \frac{\sin 2\theta}{2} \Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \left(\frac{3\pi}{2} + \frac{\sin 3\pi}{2}\right) - \left(\frac{\pi}{2} + \frac{\sin \pi}{2}\right)$$

$$= \frac{3\pi}{2} - \frac{\pi}{2}$$

$$= \pi$$