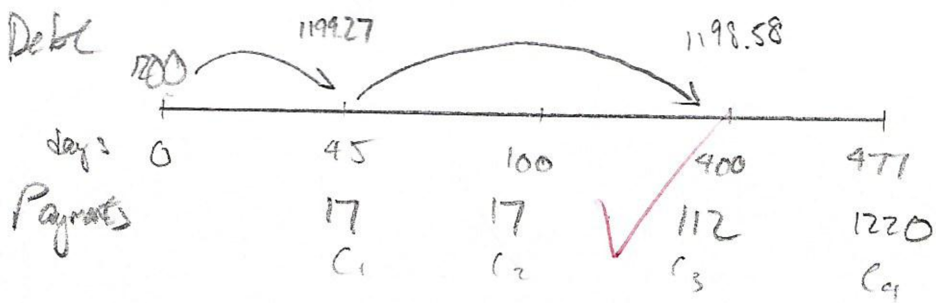
Student Number: 140589590

[8 marks] 1. Sonny borrowed \$1200 today at a simple interest rate of $r = 11\%$. He will pay it back with payments of \$17 in 45 days (from today), \$17 in 100 days (from today), \$112 in 400 days (from today) and a final payment of \$1220 in 477 days (from today).

Suppose the Declining Balance Method is in effect, what is the balance of the loan immediately after any payments have been made 400 days from the day Sunny borrowed the money (today)? Be sure to include a time diagram in your solution.



@ day 45:

$I = 1200 \cdot 11\% \cdot \frac{45}{365} = 16.27$

$16.27 < 17$

New Debt $= 1200\left(1 + 11\% \cdot \frac{45}{365}\right) - 17 = 1199.27$

@ day 100:

$I = 1199.27 \cdot 11\% \cdot \frac{55}{365} = 19.88$

$19.88 > 17$ $\qquad I > C_2$

$\therefore$ carry forward.

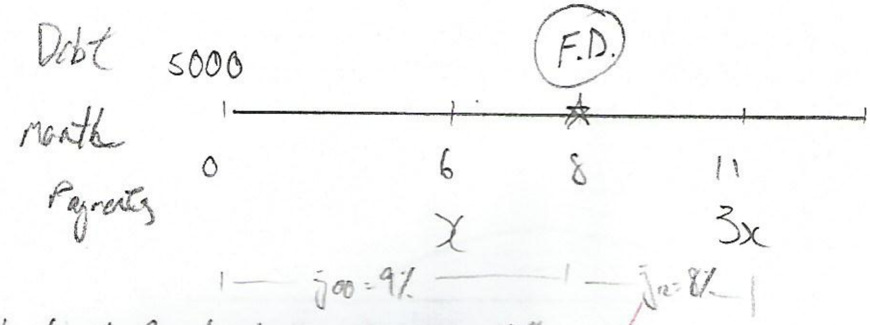
@ day 400:

$I = 1199.27 \cdot 11\% \cdot \frac{355}{365} = 128.31$

$128.31 < (17 + 112) = 129$

New Debt $= 1199.27\left(1 + 11\% \, \frac{355}{365}\right) - (112 + 17) = 1198.58$

[*6 marks*] 2. You borrow \$5000 now and agree to pay \$$X$ in 6 months from now and \$$3X$ in 11 months from now. Determine X if interest is at 9% compounded continuously for the first 8 months followed by 8% compounded monthly for the remainder of the term. Be sure to include a time diagram with all the appropriate information.



Let the focal date to be the 8th month:

$$5000\left(e^{9\%\cdot\frac{8}{12}}\right)=X\left(e^{9\%\cdot\frac{2}{12}}\right)+3x\left(1+\frac{8\%}{12}\right)^{-3}$$

$\boxed{X=\$1342.09}$

6/6

[*6 marks*] 3. Consider the following time diagram:

			\$1	\$1	\$1	\$1	\$1					\$1	\$1	\$1	\$1			
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

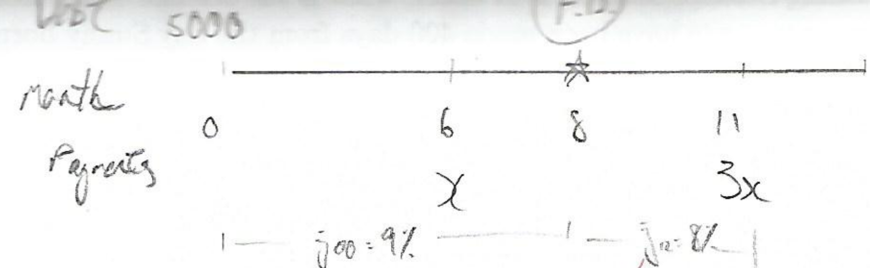
Assume the interest per payment period is i. Provide a simplified expression, using only the symbols i, $s_{\overline{n}|i}$, or $a_{\overline{n}|i}$, for a single payment X that is equivalent to all 9 payments of \$1 at times:

(a) $t = 2$

$$X = a_{\overline{5}|i} + a_{\overline{4}|i}(1+i)^{-9}$$

6/6

(b) $t = 11$



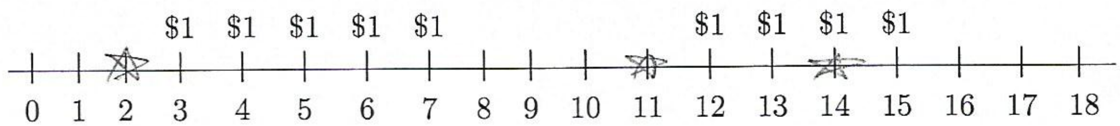
Let the focal date to be the 8^{th} month:

$$5000\left(e^{9\% \cdot \frac{8}{12}}\right) = X\left(e^{9\% \cdot \frac{2}{12}}\right) + 3x\left(1+\frac{8\%}{12}\right)^{-3}$$

$\boxed{X = \$1342.09}$

6/6

[*6 marks*] 3. Consider the following time diagram:



Assume the interest per payment period is i. Provide a simplified expression, using only the symbols i, $s_{\overline{n}|i}$, or $a_{\overline{n}|i}$, for a single payment X that is equivalent to all 9 payments of \$1 at times:

(a) $t = 2$

$$X = a_{\overline{5}|i} + a_{\overline{4}|i}(1+i)^{-9}$$

6/6

(b) $t = 11$

$$X = s_{\overline{5}|i}(1+i)^{4} + a_{\overline{4}|i}$$

(c) $t = 14$

$$X = s_{\overline{5}|i}(1+i)^{7} + s_{\overline{4}|i}(1+i)^{-1}$$

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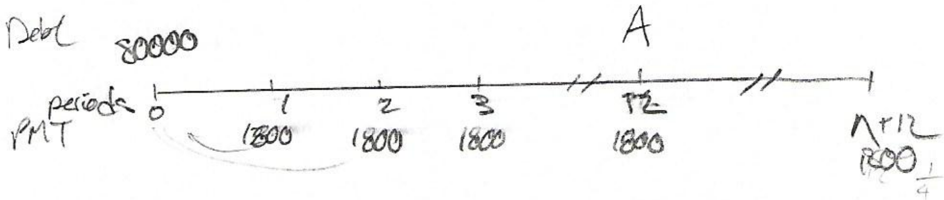
[*13 marks*] 4. Mr. Kabbes, won \$100 000 in a drumming contest. He will receive \$20 000 immediately and regular payments at the end of every 3 months thereafter. It is determined that interest will be paid at $j_4 = 6\%$ for the first 3 years and $j_4 = 7\%$ thereafter.

(a) If Mr. Kabbes recieves regular payments of \$1800 how many full payments will he receive? Be sure to include a time diagram in your solution.



$A = 80000$

$R \cdot 1800$

first 3yrs $j_4 = 6\%$

after $j_4 = 7\%$

$n = 36$

Balance after 3 years:

$$1800 \frac{1-(1+\frac{6\%}{4})^{-12}}{\frac{6\%}{4}} = 19633.51$$

↳ not full amount.

$$80000(1+\tfrac{6\%}{4})^{12} = 1800 \frac{(1+\frac{6\%}{4})^{12}-1}{\frac{6\%}{4}} + A$$

$A = 72175.27$

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$A = 72175.27$

$R = 1800$

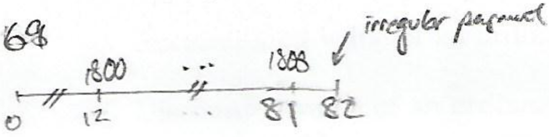
$j_4 = 7\%$

$n = ?$

$A = R \frac{1-(1+i)^{-n}}{i}$ $\quad n = \frac{\ln((1-\frac{A}{R}i)^{-1})}{\ln(1+i)}$

$n = 69.7$

$\lfloor n \rfloor = 69$



∴ He will receive 81 full payments.

(b) Rather than taking regular quarterly payments of \$1800, Mr. Kabbes determines that he can get 138 regular quarterly payments of \$1500. What payment 3 months after the last full payment of \$1500 will exhaust his winnings? Be sure to include a time diagram in your solution.

D

	80000		A					
Period	0	1	12	13	…	138	139
PMT		1500 …	1500	1500	…	1500	x

6/6

$$A = 80000\left(1+\frac{6\%}{4}\right)^{12} - 1500\,\frac{\left(1+\frac{6\%}{4}\right)^{12}-1}{\frac{6\%}{4}}$$

$$= 76087.64$$

@ 12:

$$76087.64 = 1500\, a_{\overline{126}|\frac{7\%}{4}} + x\left(1+\frac{7\%}{4}\right)^{-127}$$

@ 139:

$$76087.64\left(1+\frac{7\%}{4}\right)^{127} = 1500\, s_{\overline{126}|\frac{7\%}{4}}\left(1+\frac{7\%}{4}\right) + x$$

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$$x = 48.50$$

$\therefore$ The last payment is \$48.50