the course.

Time Allowed: 150 minutes
Total Value: 100 marks
Number of Pages: 9

Instructions:

Cheat Sheet: One 8.5" × 11" page of study notes (both sides) is allowed as a reference while completing the mock test. Please note, that the cheat sheet is permitted for the mock test only!!

Non-programmable, non-graphing calculators are permitted. No other aids allowed.

Check that your test paper has no missing, blank, or illegible pages. Note that test questions appear on ***both*** *sides of the paper.*

Answer in the spaces provided.

Show all your work. Insufficient justification will result in a loss of marks.

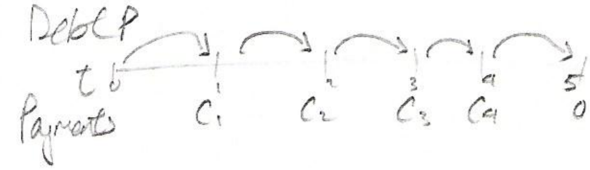
1. [6 *marks*] When repaying a debt with irregular partial payments: (a) describe the Declining Balance Method as well as the Merchant's Rule that might be used; (b) and provide a simple example with one partial payment to illustrate the difference between the two methods.

a) Declining Balance = the balance of debt will be ~~deducted~~ reduced every time payments (also covering interests) are made ✓

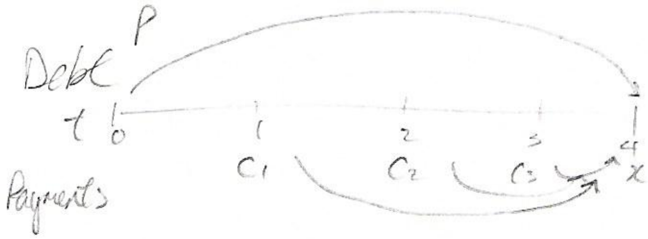
Merchant's Rule = the present value of the debt is moved forward to the maturity date, along with all interest + parti[al] payments. ✓

4/6

Declining Balance:



Merchant's Rule:



Answers:

DB: interest is calculated at each date a partial payment is made.

MR: interest is accumulated on original debt and any partial payments to the

2. [4 *marks*] Determine how long it will take \$1000 to accumulate to at least \$2500 if the investment earns interest at a continuously compounded rate of $j_\infty = 6\%$.

$n = ?$

$P = 1000$

$S = 2500$

$j_\infty = 6\%$

$$2500 = 1000\, e^{6\% \cdot n}$$

$$2.5 = e^{0.06n}$$ ✓

$$\frac{\ln 2.5}{0.06} = n$$

4/4

$0.2715 \times 12 = \underline{3.258}$

$$n \doteq 15.2715$$

$0.258 \times 30 = 7.74$

✓

$\therefore$ It will take 15 yrs, 3 months and 8 days

OR 15 years, 100 days

3. [5 *marks*] Sheldon borrows \$4000 today in return for an agreement to \$$X$ in six months, \$$X$ in 12 months and \$$2X$ in 18 months. Determine X if interest is charged at $j_2 = 7\%$ for the first year, and $j_2 = 8.5\%$ thereafter.

	0	6	12	18
D	4000			
P		X	X	$2X$

|— 3.5% —|

|— 4.25% —|

$$4000 = X(1+3.5\%)^{-1} + X(1+3.5\%)^{-2} + 2X(1+3.5\%)^{-2}(1+4.25\%)^{-1}$$ ✓

5/5

$$X = \frac{4000}{(1+3.5\%)^{-1} + (1+3.5\%)^{-2} + 2(1+3.5\%)^{-2}(1+4.25\%)^{-1}}$$

$= \underline{\$1083.83}$ ✓

4. [5 *marks*] You borrow $1600 from a licensed small loan company and agree to pay $160 a month for 12 months. Use linear interpolation to determine the nominal rate j_{12} that the company is charging.

$A = P = 1600$

$R = 160$

$n = 12$

$j_{12} = ?$

$1600 = 160\, a_{\overline{12}|i}$ ✓

$= 160 \frac{(1-(1+i)^{-12})}{i}$

$j_{12} \approx 12\left(\frac{R}{A} - \frac{A}{Rn^2}\right)$

$= 36.77\%$

$\lceil j_{12}' \rceil = 37\% \Rightarrow A = 1584.75$

$\lfloor j_{12}' \rfloor = 36\% \Rightarrow A = 1592.64$ ✓

$35\% \Rightarrow A = 1600.59$

5/5

$\frac{1600.59 - 1592.64}{35\% - 36\%} = \frac{1600 - 1592.64}{j_{12} - 36\%}$ ✓ $j_{12} = 35.07\%$ ✓

5. [20 *marks*] The Wangs purchase a new house and finance it by signing a mortgage contract for $325 000 to be repaid in monthly installments at $j_2 = 4\frac{1}{2}\%$ over 25 years.

(a) Calculate the monthly payment, rounded up to the next dollar.

$A = 325{,}000$

$n = 300$

$\left(1 + \frac{4.5\%}{2}\right)^2 - 1 = i = 0.37\%$

$R = ?$

keep more decimals for accuracy

$A = R\, a_{\overline{n}|i}$ ✓

$325000 = R\, \frac{1-(1+i)^{-300}}{i} \rightarrow R = 1798.79$

$R = 1{,}799$

give calculated value first, then determine amount with any adjustments

so will pay $1799/month

5/6

(b) At the end of 5 years, the mortgage must be renegotiated. Determine the outstanding balance on the mortgage.

Outstanding balance:

$k = 60$

(c) The mortgage is refinanced at $j_2 = 6\frac{1}{4}\%$. Determine the new monthly payment, again rounded up to the next dollar.

$B_{10} = 285323.98$

new $n = 240$ ✓

$\sqrt[12]{\left(1+\frac{6.25\%}{2}\right)^2} - 1 =$ new $i = 0.51\%$

$R = ?$

✓

$285323.98 = R\, a_{\overline{240}|i} \Rightarrow R = ?$

$R = \$2,073$

so pay \$2073/month.

3/3

(d) Instead of the amount calculated in part (c), the Wangs decide to make monthly payments of \$2500. Assuming rates remain at $j_2 = 6\frac{1}{4}\%$ for the remaining life of the mortgage, determine (i) the number of full payments to be made, and (ii) the value of the last drop payment.

i) $i = 0.51\%$
$A = 285323.98$
$R = 2500$
$n = ?$

$A = R\frac{1-(1+i)^{-n}}{i}$

$n = \frac{\ln\left[\left(1-\frac{Ai}{R}\right)^{-1}\right]}{\ln(1+i)}$ ✓

$= 172.35$

✓

at 172 payments:

Debt Balance $= 285323.98(1+i)^{172} - 2500\, s_{\overline{172}|i}$

$= \$862$

$\therefore$ 172 full payments

7/8

ii) @173 :

$285323.98(1+i)^{173} = 2500\, s_{\overline{172}|i}(1+i) + X$ ✓

$X = 866.43$

$866.43 + \cancel{2500} = \$3366.43$

last pmt is only this amount.

(e) Calculate the total interest paid on the mortgage.

$I = 325000(1+0.37\%)^{40} + 285323.98(1+0.51\%)^{172} - 325000(1+0.37\%)^{40}(1+0.51\%)^{172}$

6. [5 *marks*] A property is currently bringing in an annual income (through rent) of \$10 000 per year, payable at the beginning of each year. It is expected that the rent will increase at 4% per year, indefinitely. Determine the value of the property today, using $j_1 = 7\%$

$$\overset{10000}{\downarrow} \quad +4\% \quad +4\% \quad +4\%$$
$$t: \; 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad n$$

$n = ?$

$R_0 = 10000$, $R_2 = 10\,000(1.04)$, $R_3 = 10\,000(1.04)^2$ etc.

$S_n = ?$

$A_0 = 10\,000$

$A_1 = 10\,000(1.04)(1.07)^{-1}$

$A_2 = 10000(1.04)^2(1.07)^{-2}$

$\therefore A$ is a geometric series with $r = \frac{1.04}{1.07}$

1/5

$$A = \frac{R}{i}(1+i)$$
$$= \frac{10000}{11\%}(1.11)$$
$$= \$100909.09$$

$i = 7\% + 4\%$

$= 11\%$

Answer: $A = 10000 + 10000(1.04)(1.07)^{-1} + 10000(1.04)^2(1.07)^{-2} + \ldots$

$= \frac{t_1}{1-r} \qquad = 356666.67 \qquad t_1\left(\frac{1-r^n}{1-r}\right)$

as $n \to 500$ $\quad \frac{t_1}{1-r}$

7. [4 *marks*] A research fund is set up to pay out \$5000 semi-annually, indefinitely. Calculate the amount that is required today in a fund earning ~~j_{12}~~ j_2 $= 8\%$, if the first research award is to be given in 5 years time.

$R = 5000$

$i = 4\%$

$$A \qquad \swarrow A_0$$
$$\vdash \quad 9 \quad 10 \quad 11$$
$$5000 \quad 5000 \quad 5000$$

3/4

~~$A_0 = \frac{R}{i} = \frac{5000}{4\%} = 125000$~~

~~$A = A_0(1+i)^{-10}$~~ -9

~~$= 125000(1.04)^{-10}$~~

~~$\$84445.52$~~

Answer:

$$A \quad 0 \; \ldots \; 9 \quad 5 \quad 6 \quad 7$$
$$5000 \quad 5000 \quad 5000 \quad 5000 \quad 5000 \quad 5000$$

$$A = A_0(1+i)^{-9}$$
$$= \frac{R}{i}(1+i)^{-9}$$
$$= \frac{5000}{4\%}(1.04)^{-9}$$
$$= 87823.34$$

8. ~~[4 *marks*] Show that~~ $\frac{1}{s_{\overline{n}|i}} + i = \frac{1}{a_{\overline{n}|i}}$.

$\frac{1}{a_{\overline{n}|i}} : \frac{1-(1+i)^{-n}}{i}$

$n=?$

$R_0=10000$, $R_2=10\,000(1.04)$

$R_2=10\,000(1.04)^2$ etc.

$S_n=?$

1/5

$A=\frac{R}{i}(1+i)$

$=\frac{10000}{11\%}(1.11)$

$=\$100909.09$ x

$i=7\%+4\%$

$=11\%$

$A_0=10\,000$

$A_1=10\,000(1.04)(1.07)^{-1}$

$A_2=10000(1.04)^2(1.07)^{-2}$

∴ A is a geometric series with $r=\frac{1.04}{1.07}$

Answer: $A=10000+10000(1.04)(1.07)^{-1}+10000(1.04)^2(1.07)^{-2}+\ldots$

$=\frac{t_1}{1-r}$ $=356666.67$

$t_1\left(\frac{1-r^n}{1-r}\right)$ as $\lim_{n\to\infty}$ $\frac{t_1}{1-r}$

7. [4 *marks*] A research fund is set up to pay out \$5000 semi-annually, indefinitely. Calculate the amount that is required today in a fund earning ~~j_{12}~~ j_2 $= 8\%$, if the first research award is to be given in 5 years time.

$R=5000$

$i=4\%$

A ↙A_0

9 10 11

5000 5000 5000

~~$A_0=\frac{R}{i}=\frac{5000}{4\%}=125000$~~

~~$A=A_0(1+i)^{-10}$~~ -9

~~$=125000(1.04)^{-10}$~~

~~$=\$84445.52$~~

Answer:

0 A ... 9 5 6 7

5000 5000 5000 5000 5000 5000

$A=A_0(1+i)^{-9}$

$=\frac{R}{i}(1+i)^{-9}$

$=\frac{5000}{4\%}(1.04)^{-9}$

$=87823.34$

3/4

$a_{\overline{n}|i}=\frac{1-(1+i)^{-n}}{i}$

8. [4 *marks*] Show that $\frac{1}{s_{\overline{n}|i}}+i=\frac{1}{a_{\overline{n}|i}}$.

L.S. $\frac{1}{\frac{(1+i)^n-1}{i}}+i$

$=\frac{i}{(1+i)^n-1}+\frac{i[(1+i)^n-1]}{(1+i)^n-1}$

$=\frac{i+i(1+i)^n-i}{(1+i)^n-1}$ ✓

$=\frac{i(1+i)^n}{(1+i)^n-1}\times\frac{\frac{1}{(1+i)^n}}{\frac{1}{(1+i)^n}}$

Touch Down

7. [4 *marks*] A research fund is set up to pay out \$5000 semi-annually, indefinitely. Calculate the amount that is required today in a fund earning ~~j_{12}~~ j_2 $= 8\%$, if the first research award is to be given in 5 years time.

$R = 5000$

$i = 4\%$

A ← A_0

9 10 11

5000 5000 5000

~~$A_0 = \frac{R}{i} = \frac{5000}{4\%} = 125000$~~

3/4

~~$A = A_0(1+i)^{-10}$~~ -9

~~$= 125000(1.04)^{-10}$~~

~~$= 84445.52$~~

Answer:

A

0 A ... 9 5 6 7

5000 5000 5000 5000 5000 5000

$A = A_0(1+i)^{-9}$

$= \frac{R}{i}(1+i)^{-9}$

$= \frac{5000}{4\%}(1.04)^{-9}$

$= 87823.34$

8. [4 *marks*] Show that $\dfrac{1}{s_{\overline{n}|i}} + i = \dfrac{1}{a_{\overline{n}|i}}$.

$\frac{1}{a_{\overline{n}|i}} = \frac{1}{\frac{1-(1+i)^{-n}}{i}}$

$LS.\ \frac{1}{\frac{(1+i)^n-1}{i}} + i$

$= \frac{i}{(1+i)^n-1} + \frac{i[(1+i)^n-1]}{(1+i)^n-1}$

$= \frac{i + i(1+i)^n - i}{(1+i)^n-1}$

✓

$= \frac{i(1+i)^n}{(1+i)^n-1} \times \frac{\frac{1}{(1+i)^n}}{\frac{1}{(1+i)^n}}$

Touch Down

9. [10 *marks*] A manufacturing company borrows \$2 000 000 to purchase a new machine. The debt is charged interest at $j_{12} = 8\%$. At the same time, the company establishes a sinking fund earning interest at $j_{12} = 5\%$ to repay the debt in 20 years.

(a) Determine the total monthly expense of the debt.

$A = 2000000$

$n = 240$

$i_{LOAN} = \frac{8\%}{12}$

$i_{SAVE} = \frac{5\%}{12}$

$2000000 = R\, s_{\overline{240}|\frac{8\%}{12}}$ use 5% here.

$R = 3395.47$ ×

↳ pmt to sinking fund.

interest pmt on loan = ?

$2000000 = R\, s_{\overline{240}|\frac{5\%}{12}}$

$R = 4865.78$

Interest:

$2000000\left(\frac{0.08}{12}\right)$

$= \$13333.33$

Total monthly expense = \$18,199.11

(b) Construct the first four lines of the sinking fund schedule, based on the monthly deposits.

# of Coupon	Coupon	Interest	Book Value Adjusted	Outstanding Balance.
0	/	/	/	0
1	4865.78	/	4865.78	4865.78
2	4865.78	20.27	4886.05	9751.83
3	4865.78	40.63	4906.41	14658.24
4	4865.78	61.08	4926.86	19585.10

take out →

(c) Determine the book value of the company's debt at the beginning of the 10th year.

$B_{108} =$

$2000000 - 4865.78\; s_{\overline{108}|\frac{5\%}{12}}$

10. [10 *marks*] Mrs. Balfour buys a \$1000 bond that pays coupons at $j_2 = 7\%$ and is redeemable at par in 15 years. The price she pays will give her a yield of $j_2 = 8\%$ if held to maturity. After 5 years, Mrs. Balfour sells this bond to Dr. Kilgour, who wants a yield of $j_2 = 6\%$ on his investment.

(a) Determine the price that Mrs. Balfour paid for the bond.

$F = 1000 = C$

$r = 3.5\%$

$n = 30$

$i = 4\%$

$P = 1000 + (1000(3.5\% - 4\%)) \times \frac{1-(1.04)^{-30}}{0.04}$

$= \$913.54$ ✓

5 15

3/4

(b) Determine the price that Dr. Kilgour paid.

$i = 3\%$ $(r = 3.5\%)$

$n = 20$

$F = 1000 = C$

$P = 1000 + (1000 \times (0.5\%)) \times \frac{1-1.03^{-20}}{0.03}$ ✓ $= \$1074.39$

$= \$1098.00$ calculation error

$= \$1074.39$

(c) Estimate the yield rate j_2 that Mrs. Balfour realized. Use method of averages, followed by linear interpolation.

Method of Avg. $i' = \dfrac{\frac{nFr + C - P}{n}}{\frac{P + C}{2}}$

P $(1098) = \frac{913.54 \; C}{(1+i)^{10}} + 1000 \cdot 4\% \cdot a_{\overline{10}|i}$

$n = 10$

$i = ?$

$(r = 4\%)$ $r = 3.5\%$ still

$F = 1000$

$P = 913.54$ $C = 1098$

0.058

$j^2 = 11.67\%$

✓ consistent with (b)

2/6

$\lceil j^2 \rceil = 12\% \Rightarrow A = ?$

$\lfloor j^2 \rfloor = 11\% \Rightarrow P = ?$

use here to get a P and s below and above 913.54

$913.54 = \frac{1074.39}{(1+i)^{10}} + 1000(3.5\%)(a_{\overline{10}|i})$

$i' = \frac{nFr + C - P}{n}$

$P = 1000 + (1000(\ldots)) \times \frac{1-\ldots}{0.04}$

$= \$913.54$ ✓

(b) Determine the price that Dr. Kilgour paid.

3/4

$i = 3\%$ ($r = 3.5\%$)

$n = 20$

$F = 1000 = C$

$P = 1000 + (1000 \times (0.5\%)) \times \frac{1 - 1.03^{-20}}{0.03}$ ✓ $= \$1074.39$

$= \$1098.00$ ← calculation error

$= \$1074.39$

(c) Estimate the yield rate j_2 that Mrs. Balfour realized. Use method of averages, followed by linear interpolation.

Method of Avg. $i' = \frac{\frac{nFr + C - P}{n}}{\frac{P + C}{2}}$

$= 0.058$

$j'_2 = 11.6\%$

2/6

$P\ 1098 = \frac{913.54\ C}{(1+i)^{10}} + 1000 \cdot 4\% \cdot a_{\overline{10}|}$

$n = 10$ $r = 4\%$ $r = 3.5\%$

$i = ?$ $F = 1000$

$P = 913.54$ $C = 1098$

✓ consistent with (b)

$\lceil j'_2 \rceil = 12\% \Rightarrow P = ?$

$\lfloor j'_2 \rfloor = 11\% \Rightarrow P = ?$

use here to get Prices below and above 913.54

$913.54 = \frac{1074.39}{(1+i)^{10}} + 1000(3.5\%)\ (a\ldots$

$i' = \frac{\frac{nFr + C - P}{n}}{\frac{P + C}{2}}$

$= 5.64\%$

$j_2 = 11.27\%$

$\frac{929.84 - 892.80}{10\% - 11\%} = \frac{913.54 - 892.80}{j_2 - 11\%}$

$\lceil j'_2 \rceil = 12\% \rightarrow P_1 = 857.54$

$\lfloor j'_2 \rfloor = 11\% \rightarrow P_2 = 892.80$

$j_2 = 10.44\%$

11. [5 *marks*] A $2000 bond, redeemable at 102 and paying coupons at $j_2 = 6\frac{1}{2}\%$ is purchased for $2072.40, which gives a yield rate of 6%. Construct the first five rows of the bond schedule.

Time	Coupon	Interest on Book Value	Book Value Adjustment	Book Value
0				2072.40
1	65	62.17 61.27	-3.73	2068.67
2	65	61.16	3.84	2064.83
3	65	61.04	3.96	2060.87
4	65	60.93	4.07	2056.80

4/5

✓method

negative to show adjustment is a decrease in value (sometimes, will be an increase)

12. [11 *marks*] A $5000 bond pays interest at $j_2 = 8\%$ and matures at par on November 1st, 2016.

(a) Determine the price paid for the bond if it was sold on July 28th, 2004, at a market quotation of 89.38.

May 1, 2004 Jy 28, 04 N 1, 16

121 $n = 25$ 209

$F = C = 5000$

$n = 25$

$r = 4\%$

$q = 89.38$

$Q = \frac{Fq}{100}$ ✓

$= 4469$

$Q = P - I$

$P = Q + I$

$= 4469 + kFr$

$= 4469 + \frac{209-121}{305-121}$ ✓ $5000 \times 4\%$

$= \$4564.65$ ✓

4/4

(b) What should the market quotation of this bond be on July 28th, 2004, to yield the buyer $j_{12} = 7\%$?

$n = 25$ ✓

$r = 4\%$

$i = \sqrt[6]{(1+\frac{.07}{12})^{12}} - 1$

$= 3.557$ ✓

$F = C = 5000$

$P_0 = 5367.59$ 63 ? show calculation... maybe round error?

6/7

$Q = P - I$ ✓ $k = \frac{88}{184}$

$= 5457.93 - \frac{11}{23} \times 5000 \times 4\%$

$= 5362.28$

$q = \frac{5362.28}{5000} \times 100$ ✓

13. [5 *marks*] A $1000 bond has semi-annual coupons at $j_2 = 7\%$. The bond matures after 20 years at par, but can be called after 15 years at $1050. Determine the price to guarantee a yield of $j_2 = 8\%$.

$F = C = 1000$

$r = 3.5\%$

$n = 40$

@ 15yr → then $C = 1050$

$P = 1000 + (1000 \times 3.5\% - (1000) \times 4\%) \times a_{\overline{30}|4\%}$

$= 913.54$

$\boxed{\text{correct} = 928.96}$

4/5

$\boxed{P = \$901.34}$

@ 20yr

$P = 1000 + 1000 \times -0.5\% \times a_{\overline{40}|4\%}$

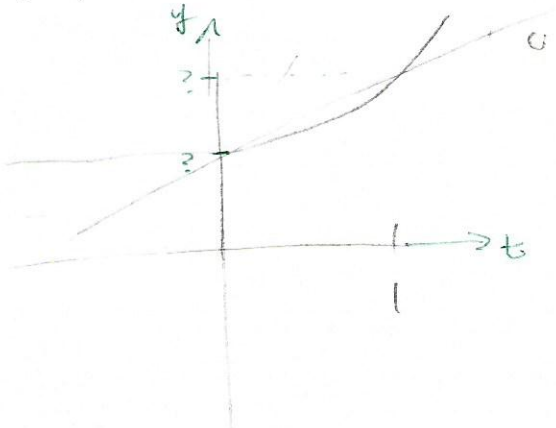
$= 901.34$ ✓

14. [10 *marks*] A $1000 bond with coupons at $j_2 = 10\%$ is redeemable at par in n years. It is purchased at a premium of $300. Another $1000 bond with coupons at $j_2 = 8\%$ is also redeemable at par in n years. It is purchased at a premium of $100 on the same yield basis as for the first bond. [That is, both n and i are the same for both bonds.]

(a) Calculate the unknown yield rate, j_2.

(b) Using your answer to part(a), determine n.

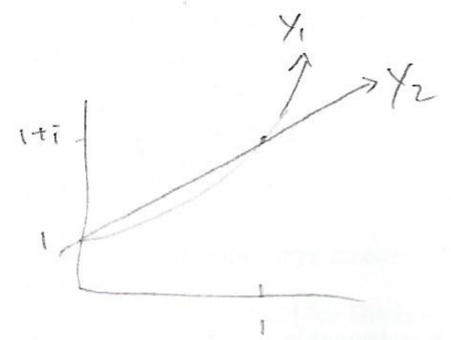
8. [4 *marks*] Assuming that $0 < i < 1$, prove that $(1+i)^t < 1 + it$ if $0 < t < 1$ by graphing $y_1 = (1+i)^t$ and $y_2 = 1 + it$ on the axis provided below. Explain the financial significance of this inequality.



← which is which? Label the graphs!

$(1+i)^t < 1+it$

1/4



Y_1 → compound interest

Y_2 → simple interest

∴ for amount invested less than y_1, simple interest will accumulate larger values.

14. [8 *marks*] Develop the two purchase-price formulae of a bond.

(a) $P = Fra_{\overline{n}|\,i} + C\,(1+i)^{-n}$

Fr = coupon payment amount

$a_{\overline{n}|i}$ = annuity of coupon over n periods, with yield rate of i

$C(1+i)^{-n}$ = present value of maturity value

be more specific, but right idea

5/8

- sum of annuity & coupon

Premium/Discount

(b) $P = C + (Fr - Ci)\,a_{\overline{n}|\,i}$

It's the maturity value of the bond adding the difference in book value adjustment