[04] 1. Write each of the following statements using the form "If........ then......":

a) x only if y If x then y

b) A unless not B If B then A

c) D whenever E If E then D

d) F is necessary for G. If G then F

e) H is sufficient for J If H then J

[04] 2. Consider the proposition "All Computer Science students take Comp 232 and Comp 233." Let $C(x)$ represent student x is in Computer Science, $M(x)$ represents x takes Comp 232, S(x) represents x takes Comp 233. Domain is all students:

a) Write the negation of the above proposition using symbols only

$\neg \forall x : \; C(x) \rightarrow M(x) \wedge S(x)$

b) Use one of DeMorgan's rules for quantifiers to write an alternate form of answer a).

$\exists x : \neg \left[C(x) \rightarrow M(x) \wedge S(x) \right]$

c) Write the answer b) without using the symbol $\rightarrow$.

$\exists x : \neg \left[\neg C(x) \vee (M(x) \wedge S(x)) \right]$

d) Use one of DeMorgan's rules to write answer c) in simplest form

$\exists x : \; C(x) \wedge \neg \, (M(x) \wedge S(x))$

e) Write answer d) in English

"There Exists a C.S. student who has NOT taken Comp 232 AND Comp 233"

[04] 3. (i) $R(x, y)$ represents $x - 5y = xy$, where x and y are integers

(ii) $Q(m, n)$ represents $m \geq n$, where the domain is the set of non negative integers.

(iii) $P(x, y)$ represents a predicate where the domain for the variables x and y is $\{1, 2, 3\}$. $P(1, 3), P(2, 1), P(2, 2), P(2, 3), P(3, 1), P(3, 2)$ are true, and $P(x, y)$ is false otherwise.

Determine the truth value of each statement and justify yor answer:

a) $\exists x \exists y \; : [P(x, y) \wedge \neg P(y, x)]$. TRUE

Choose
$x = 2$
$y = 1$

b) $\forall x \exists y \; : R(x, y)$. FAlse

$x - 5y = xy$
$\Rightarrow x = xy + 5y$
$\Rightarrow x = y(x+5)$
$\Rightarrow y = \frac{x}{x+5}$ $\Rightarrow$ DNE when $x = -5$

c) $\exists n \forall m \; : Q(m, n)$. true

Choose $n = 0$

Now all non negative integers $m \geq 0$.

[05] 4. Ali attends the concert but Ed does not. Ali does not attend or Jill does not attend implies Kelly does not attend. Kelly attends or Ed does not attend. It is not true that Ali and Jill attend..

a) Using the symbols A = Ali, E = Ed, J = Jill, K = Kelly symbolize the four given statements and simplify if possible.

① $A \wedge \neg E$

② $(\neg A \vee \neg J) \to \neg K$

③ $K \vee \neg E$

④ $\neg(A \wedge J) \equiv \neg A \vee \neg J$

b) Determine, if possible, who attended the concert and who did not attend the concert. Give the final answer in English but explain your reasoning using the symbols.

Assume $A = T \Rightarrow \neg A = F$

$\Rightarrow$ $\neg E = T \Rightarrow E = F$ by ①

Now $\neg J = T \Rightarrow J = F$ by ④

Now $(\neg A \vee \neg J) = T \Rightarrow \neg K = T \Rightarrow K = F$ by ②

Also note $K \vee \neg E$ is true in ③

$\Rightarrow A = T,\ E = F,\ J = F,\ T = F$

Satisfy all 4 Conditions

Hence Ali Attends but Ed, Jill, Kelly do not Attend

OR

[05] 5. Determine whether the following statement is a tautology, contradiction or neither using logical equivalences without a truth table: $[\neg p \to (q \to r)] \to [q \to (p \vee r)]$

$[\neg p \to (q \to r)] \to [q \to (p \vee r)]$	
$\equiv p \vee (q \to r) \to \neg q \vee (p \vee r)$.	Implication in terms OR
$\equiv p \vee (\neg q \vee r) \to \neg q \vee p \vee r$	" " "
$\equiv (\neg q \vee p \vee r) \to (\neg q \vee p \vee r)$	Commutative, Associative
$\equiv \neg(\neg q \vee p \vee r) \vee (\neg q \vee p \vee r)$	Implic. in terms OR
$\equiv T$	$\neg x \vee x \equiv T$
$\Rightarrow$ Tautology	DEF of tautology

[08] 6. For each statement in question 6 state whether it is True or False. If True prove by Contraposition, if False prove by showing a Counter example:

a) $\forall x \in R \quad x^2 > x$

This statement is False

Proof: (Counter Example)

Let $x = \frac{1}{2}$

then $x^2 = (\frac{1}{2})^2 = \frac{1}{4}$

3 marks

$\Rightarrow x^2 > x$ is False

QED.

Can choose any $0 \leq x \leq 1$

Algebra

At least when $x = \frac{1}{2}$

b) Consider $P(n) = n^2 + 7$. $\forall n \in Z$: If $P(n)$ is odd then n is even.

Proof:

(Contraposition)

Form the Contrapositive:

If n is odd then $P(n)$ is even

Consider n odd.

$\Rightarrow n = 2k+1$ where $k \in Z$ — Def of odd Integer

5 marks

$\Rightarrow P(n) = n^2+7 = (2k+1)^2+7$ — Substitute

$\Rightarrow P(n) = 4k^2+4k+1+7$

$\Rightarrow P(n) = 4k^2+4k+8$

$\Rightarrow P(n) = 2(2k^2+2k+4)$

$\Bigg\}$ Algebra

Since $(2k^2+2k+4) \in Z$ — Z is closed for Add & mult.

$\Rightarrow P(n)$ is even — Def. of Even Integer

Since If n is odd then $P(n)$ is even

is a True statement

$\Rightarrow$ If $P(n)$ is odd then n is even

is also true

Original $\equiv$ Contrapos.

[05] 7. The equation $9x^2 + y^2 = 8$ has no positive integer solutions. Give a proof by cases of this statement.

Proof: (by Cases)

Step 1 Reduce the possible x, y	
① No need to test negative x, y	by given
② No need to check $x=0, y=0$	by given
Step 2 Case $x \geq 1$ AND $y \geq 1$ $(x, y \in Z^+)$	only 1 case
$x \geq 1 \Rightarrow 9x^2 \geq 9$	Alg.
$y \geq 1 \Rightarrow y^2 \geq 1$	Alg.
$\Rightarrow 9x^2 + y^2 \geq 9 + 1 = 10$	Substitute

$\Rightarrow \forall x \forall y \; x \geq 1, y \geq 1 : 9x^2 + y^2 \geq 10$

$\Rightarrow \forall x \forall y \; x \geq 1, y \geq 1 : \neg(9x^2 + y^2 < 10)$

$\Rightarrow \neg \exists x \exists y, x \geq 1, y \geq 1 : (9x^2 + y^2 < 10) \Rightarrow 9x^2 + y^2 = 8$ has No Int. Sol.

[05] 8. $\forall x \forall y \in R$: If x is irrational and y is rational then $(x - y)$ is irrational. Give a proof by contradiction of this statement.

Proof: (Contradiction)

Step 1 Either $(x-y)$ is Irrational is T OR $(x-y)$ is Irrational is F	List all possibilities
Step 2 Assume $(x-y)$ is Irrational is F	Assume poss. you do NOT want
$\Rightarrow (x-y)$ is Rational	
$\Rightarrow x - y = \frac{a}{b}$ where $a, b \in Z, b \neq 0$	Def of Rational
$\Rightarrow x = \frac{a}{b} + y$	Algebra
Since y is Rational	Given
$\Rightarrow y = \frac{c}{d}$, where $c, d \in Z, d \neq 0$	Def of Rational
Now Substitute for y	
$\Rightarrow x = \frac{a}{b} + \frac{c}{d}$	
$\Rightarrow x = \frac{ad + bc}{bd}$	Common Den.
Now $(ad + bc), bd \in Z$	Close for Add, Mult. in Z
$\Rightarrow x$ is Rational	Def of Rational
Contradicts the given	
Step 3 $\Rightarrow (x-y)$ is Irrational	only remaining possibility.

QED

[05] 9. Consider the following decision table whose input specifications are the Boolean variables x, y, z. The Conjunction of the x, y, z values in each each row form value F for that row.

SPECIFICATIONS

x	y	z	F
1	1	0	$xy\bar{z}$
1	1	1	xyz
1	0	1	$x\bar{y}z$
0	0	1	$\bar{x}\bar{y}z$

a) Fill in the above blanks with the value of F in each row.

b) Form the Boolean expression R which is the Disjunction of the above four F values

$$R = xy\bar{z} + xyz + x\bar{y}z + \bar{x}\bar{y}z$$

c) Simplify the answer for R using Boolean Algebra. Answer in terms of x, y, z

$$R = xy(\bar{z}+z) + \bar{y}z(x+\bar{x})$$

$$= xy(1) + \bar{y}z(1)$$

$$R = xy + \bar{y}z$$

d) Using the simplified version of R complete the folowing statement in English:
R is true when x is true and y is true OR y is false and z is true.

[02] Bonus From the following approaches state all that are valid if you are asked to prove LHS $\leftrightarrow$ RHS:

a) Prove: [LHS $\rightarrow$ RHS] and [RHS $\rightarrow$ LHS] ✓

b) Prove: [LHS $\rightarrow$ RHS] and [$\neg$ RHS $\rightarrow$ $\neg$ LHS] ✗ (these are ≡, RHS→LHS missing)

c) Prove: [$\neg$ LHS $\rightarrow$ $\neg$ RHS] and [$\neg$ RHS $\rightarrow$ $\neg$ LHS] ✓

d) Prove: [RHS $\rightarrow$ LHS] and [$\neg$LHS $\rightarrow$ $\neg$RHS] ✗ (these are ≡, LHS→RHS missing)

total

40 *marks*

$\left.\begin{array}{l}\text{Approach a)}\\ \text{Approach c)}\end{array}\right\}$ Are valid