

Question 1 (30 points)

Answer the following short questions, 5 points each.

- 1-1. A bond carries 8% annual coupon was issue on July 2005 and will mature on July 31, 2015. It pays interests semiannually. You have bought the bond with face value of \$100,000 on September 15, 2011. Specify the amount and time of the cash flows you will receive until maturity.

$$C = 8\% \times 100,000 = \frac{8000}{2} \text{ per month} = 4000 \checkmark$$

From July 2005 \$4000

January 2006 \$4000

July 2006 \$4000 ✓

July 31 2015 \$104000 ✓

- 1-2. The interest rate is quoted as 10% APR based on semiannual compounding assumption. What is the interest rate per month? What should the interest rate be quoted as APR based on monthly compounding?

$$EAR = \left(1 + \frac{10\%}{2}\right)^2 - 1$$

$$= (1 + 0.05)^2 - 1$$

$$= 0.1025 \text{ OK}$$

$$1 + EAR = \left(1 + \frac{0.1}{2}\right)^2 = 1.1025$$

$$r_m = \sqrt[12]{1.1025} - 1$$

$$(1 + r_m)^{12} = EAR + 1 = 1.1025$$

$$r_m = \sqrt[12]{1.1025} - 1 \text{ OK}$$

$$= 0.008165 \text{ OK}$$

$$\left(1 + \frac{APR}{n}\right)^n = (1 + r_m)^m$$

$$APR = 12 \times r_{12}$$

$$= 0.098 \text{ or } 9.8\%$$

$$\left(1 + \frac{10\%}{2}\right)^2 = (1 + r_{12})^{12}$$

$$r_{12} = 0.8165\%$$

1-3 A bond market commentator comments that "the bond market tumbled today after the announcement of the central bank". What is likely the content of the announcement on the interest rate? Why?

5  
If the interest rate decreases, the bond will increase. Everyone hopes the bond will increase, then the Canadian dollar will increase. However, the bank is unlikely to increase the interest rate. Because interest increase, prices will decrease, there will be inflation.

1-4. What are the two primary factors in determining whether funds will be lent to an applicant for a mortgage loan?

The two main primary factors determining whether funds will be lent to an applicant for a mortgage loan are PTT (pay to income) → monthly payment to check the bank's  
LTV (loan to value) → to check the lender's property

1-5. Suppose that you are reviewing a price sheet for bonds and see the following prices (per \$100 par value) reported. You observe what seem to be several errors. Without calculating the price of each bond, indicate which bonds seem to be reported incorrectly, and explain why.

Bond	Price	Coupon Rate (%)	Required Yield (%)
✓ U	90	6	< 9
V	96	9	> 8
✓ W	110	8	6
X	105	0	< 5
Y	107	7	< 9
✓ Z	100	6	6

Bond V, X, Y are wrong.

If Required Yield > Coupon Rate For example: Bond U then Price < par value (100)

If Required Yield < Coupon Rate For example: Bond W then Price > par value (100)

If Required Yield = coupon Rate the price = par value (100) For example: Bond Z

1-6. A pension fund manager knows that the following liabilities must be satisfied:

Years from Now	Liability (in millions)
1	2.0
2	3.0
3	5.4
4	5.8

Suppose that the pension fund manager wants to invest a sum of money that will satisfy this liability stream. Assuming that any amount that can be invested today can earn an annual interest rate of 7.6%, how much must be invested today to satisfy this liability stream? Be accurate to cent.

$r = 7.6\%$

Years = 4  
 $n \times 4$

Years

1	$2 \times 0.076 = 0.152$ Millions
2	$3 \times 0.076 = 0.228$ Millions
3	$5.4 \times 0.076 = 0.4104$ Millions
4	$5.8 \times 0.076 = 0.4408$ Million.

$$P = \frac{1 - \frac{1}{(1+r)^n}}{r}$$

$$PV = \frac{5.8}{(1+0.076)^4} + \frac{5.4}{(1+0.076)^3} + \frac{3}{(1+0.076)^2} + \frac{2}{(1+0.076)^1}$$

$$= \frac{1 - \frac{1}{(1+0.076)^4}}{0.076} = \frac{1 - \frac{1}{1.3404}}{0.076} = \frac{1 - 0.746}{0.076} = 3.34$$

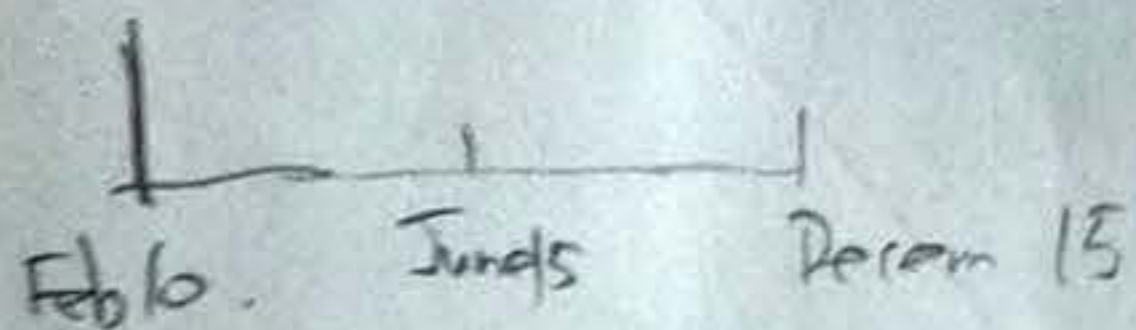
17 Question 2 (20 points)

A U.S. Treasury bond carries 4.5% annual coupon and matures on June 15, 2020. Remember that U.S. Treasury bonds pay semiannual coupon, and bond yield is quoted as annual percentage rate (APR) based on semiannual compounding.

- (a) (10 points) The bond yield on December 15, 2010 is 2.5%. What is the bond price quote?  
 (b) (10 points) Suppose that today is Feb 10, 2011. You want to sell the bond today to yield 2.4%. How should you quote it? For simplicity, assume that the trade can be settled on the same day.

chapter 2.

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$$C = \frac{4.5\% \times 100}{2} = 2.25$$

$$h = 21$$

$$r = 2.5\%$$

$$r_s = 0.0125$$

2020  
2010. 20  
10.

u =

1.298  
0.7704

$$a) P_s = 2.25 \left[ \frac{1 - \frac{1}{(1+r_s)^n}}{r} \right] + \frac{F}{(1+r_s)^n} \quad n=19 - 1$$

$$= 2.25 \left[ \frac{1 - \frac{1}{(1+0.0125)^{21}}}{0.0125} \right] + \frac{100}{(1+0.0125)^{21}}$$

$$= 2.25 \times \left[ \frac{0.2296}{0.0125} \right] + \frac{100}{1.29806}$$

$$= 41.328 + 77.038$$

$$= 118.366 \times$$

$$b) u = \frac{16 + 31 + 10}{182} = 0.3132$$

$$r_{s\text{new}} = \frac{0.024}{2} = 0.012$$

$$AI = \frac{C}{2} \times u$$

$$= 2.25 \times 0.3132$$

$$= 0.7047$$

$$P = (1 + r_{s\text{new}})^u \cdot P_s$$

$$= (1 + 0.012)^{0.3132} \times 118.366$$

$$= (1.012)^{0.3132} \times 118.366$$

$$= 118.809 \times$$

$$5 \quad P_{\text{quoted}} = P_I - AI$$

$$= 118.809 - 0.7047$$

$$= 118.1043 \times$$

16  
9

14  
8

**Question 3 (20 points)**

For simplicity, the yield curve in this question is always flat, which means that the spot rates are always equal regardless maturity, but the level can change over time. Assume that the current interest rate level is 3% (BEY). You expect that, as the economy continues its recover course, the Federal Reserve is like to increase the interest rate at a pace of 50 basis points every 6 months for the next 2 years. You have a two-year investment horizon. If you buy a U.S. Treasury bond that carries 4% annual coupon with 8 years left to maturity, what is your annualized holding period return (in BEY) over your investment horizon?

years	BEY	rs	# of left years	# of period	Price
0	0.03	0.015	8	16	$P_0$
0.5	0.035	0.0175	7.5	15	
1	0.04	0.02	7	14	
1.5	0.045	0.0225	6.5	13	
2	0.05	0.025	6	12	$P_2$

Coupon %	Interest	CI
4	0	0
4	2	
4	2	
4	2	
4	2	$CI_2$

$$\frac{C}{2} = \frac{4\% \times 100}{2} = 2$$

$$P_0 = 2 \times PVIFA(0.015, 16) + \frac{100}{(1+0.015)^{16}}$$

$$= 2 \times \frac{1 - \frac{1}{(1+0.015)^{16}}}{0.015} + \frac{100}{1.268985548}$$

$$= 2 \times \frac{1 - 0.78803}{0.015} + 78.8031$$

$$= 2 \times 14.1313 + 78.8031$$

$$= 107.0658 \checkmark$$

$$CI_2 = 2(1+0.0175)^3 + 2(1+0.02)^2 + 2(1+0.0225) + 2$$

$$= 2.107 + 2.081 + 2.05 + 2$$

$$= 8.238 \checkmark$$

$$P_2 = 2 \times \frac{1 - \frac{1}{(1+0.025)^{12}}}{0.025} + \frac{100}{(1+0.025)^{12}}$$

$$= 2 \times \frac{1 - \frac{1}{1.34489}}{0.025} + \frac{100}{1.34489}$$

$$= 2 \times \frac{1 - 0.74356}{0.025} + 74.3556$$

$$= 2 \times 10.2576 + 74.3556$$

$$= 94.8708 \checkmark$$

$$\text{Horizon Return} = \sqrt[n]{\frac{P_n + CI}{P_0}} - 1 = \sqrt[4]{\frac{94.8708 + 8.238}{107.0658}} - 1 = 0.9906 - 1$$

$$= -0.00937053$$

$$\text{BEY} = 2 \times -0.00937053 = -0.018741 \checkmark$$

Question 4 (20 points)

You have just borrowed a mortgage loan of \$200,000 from CIBC with a fixed rate of 4% for two years (the rate is quoted as BEY). You choose monthly payment and 25 year amortization.

- (1) (5 points) What is your monthly payment?
- (2) (7 points) What is the interest portion of your 20<sup>th</sup> monthly mortgage payment?
- (3) (8 points) Suppose that the two-year mortgage rate after two years rises to 5% (BEY), what will your monthly payment be after renewal?

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$$r = \frac{4\%}{12} = 0.0033 \quad r = \frac{6\sqrt{1.02} - 1}{12}$$

$$25 \times 12 = n = 300$$

$$1) \quad \text{PMT} = \text{MB} \times \frac{r(1+r)^n}{(1+r)^n - 1}$$

$$= 200,000 \times \frac{0.0033(1.0033)^{300}}{(1.0033)^{300} - 1}$$

$$= 200,000 \times \frac{0.0033(2.68685)}{1.686852}$$

$$= 200,000 \times 0.005256$$

$$= 1051.2606 \quad \text{OK}$$

$$r_{\text{new}} = \frac{0.05}{12} = 0.004167$$

$$2) \quad \text{PMT} = \text{MB} \times \frac{r(1+r)^n}{(1+r)^n - 1} \quad n=20$$

$$200,000 \times \frac{0.0033(1.0033)^{20}}{(1.0033)^{20} - 1} = 1035.242$$

Mortgage Balance at the end of Month 24 = 1052.04 + 182.8337 = 192348.46

$$3) \quad 2 \times 12 = 24$$

$$\text{new } n = 300 - 24 = 276$$

Interest portion of the 20th payment = 0.00330589 \* 192348.46 = 635.88

$$\text{MB}_+ = \text{PMT}_{\text{previous}} \times \text{PVIFA}(r, n-t)$$

$$= 1051.2606 \times \frac{1 - \frac{1}{(1+0.0033)^{276}}}{0.0033}$$

$$= 1051.2606 \times \frac{1 - \frac{1}{2.4826}}{0.0033}$$

$$= 1051.2606 \times 180.969$$

$$= 190245.58 \quad \text{OK}$$

$$\text{PMT}_{\text{new}} = 190245.58 \times \frac{0.004167(1.004167)^{276}}{(1.004167)^{276} - 1}$$

$$= 190245.58 \times \frac{0.004167(3.151)}{3.151 - 1}$$

$$= 190245.58 \times \frac{0.01313}{2.151}$$

$$= 1161.2852$$

$$r = \frac{6\sqrt{1.025} - 1}{12}$$