

Assignment 1: Electric Current Resistance and Circuits

Assigned: Jan 17th

Due: Jan 24th 6:00PM

1. An electric current is given by the expression $I(t) = 100 \sin(120\pi t)$, where I is in amperes and t is in seconds. What is the total charge carried by the current from $t = 0$ to $t = (1/240)$ s?

$$Q = \int_{t=0}^{t=1/240} I(t) dt = \int_{t=0}^{t=1/240} 100 \sin(120\pi t) dt = -\frac{100}{120\pi} [\cos(120\pi t)]_0^{1/240} = -\frac{100}{120\pi} \left[\cos \frac{\pi}{2} - \cos 0 \right] = \frac{5}{6\pi}$$

2. 1 m long copper wire of 1mm diameter is stretched permanently to 1.5 m length. What is its new resistance ?

$$R_1 = \rho \frac{l_1}{A_1} \quad R_2 = \rho \frac{l_2}{A_2} \quad V_1 = V_2 \Rightarrow l_1 A_1 = l_2 A_2$$

$$R_1 = \rho \frac{l_1}{A_1} \quad R_2 = \rho \frac{1.5l_1}{A_1/1.5} = (1.5)^2 \rho \frac{l_1}{A_1} = 2.25R_1 = 2.25(1.68 \cdot 10^{-8}) \frac{1}{\pi(0.0005)^2} \Omega = \frac{3.78}{78.5} \Omega = 0.048\Omega$$

3. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

$$I = \frac{P}{\Delta V} = \frac{600 \text{ W}}{120 \text{ V}} = \boxed{5.00 \text{ A}} \quad \text{and} \quad R = \frac{\Delta V}{I} = \frac{120 \text{ V}}{5.00 \text{ A}} = \boxed{24.0 \Omega}.$$

4. A toy battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R . (a) What is the value of R ? (b) What is the internal resistance of the battery?

$$\Delta V_{term} = \Delta V_{load} \Rightarrow P_R = I \Delta V_{term} \Rightarrow 20 \text{ W} = I(11.6 \text{ V}) \Rightarrow I = \frac{20}{11.6} \text{ (A)} = 1.72 \text{ A}$$

$$\Delta V_{load} = IR \Rightarrow R = \frac{\Delta V_{load}}{I} = \frac{(11.6)^2}{20} = 6.73 \Omega$$

$$rI = \varepsilon - \Delta V_{term} \Rightarrow r = \frac{\varepsilon - \Delta V_{term}}{I} = \frac{3.4}{20/11.6} = \frac{(3.4)(11.6)}{20} = 1.97 \Omega$$

5. How much energy is dissipated as heat during a two-minute time interval by a 1.5-k Ω resistor which has a constant 20-V potential difference across its leads?

- a) 58 J b) 46 J c) 32 J d) 72 J e) 16 J

$$P_R = \frac{(\Delta V_{load})^2}{R} ; \quad E = P_R \Delta t = \frac{(\Delta V_{load})^2}{R} \Delta t = \frac{20^2}{1500} 120 = \frac{480}{15} = 32 \text{ J}$$

Show your work

ASSIGNMENT 1: CONT.

6 Calculate the power delivered to each resistor in the circuit shown in Figure (right):

$$R_p = \left(\frac{1}{3.00} + \frac{1}{1.00} \right)^{-1} = 0.750 \Omega$$

$$R_s = (2.00 + 0.750 + 4.00) \Omega = 6.75 \Omega$$

$$I_{\text{battery}} = \frac{\Delta V}{R_s} = \frac{18.0 \text{ V}}{6.75 \Omega} = 2.67 \text{ A}$$

$$P = I^2 R : P_2 = (2.67 \text{ A})^2 (2.00 \Omega) \quad P_2 = \boxed{14.2 \text{ W}} \text{ in } 2.00 \Omega$$

$$P_4 = (2.67 \text{ A})^2 (4.00 \Omega) = \boxed{28.4 \text{ W}} \text{ in } 4.00 \Omega$$

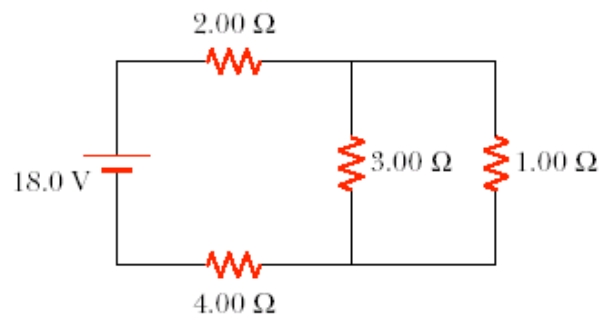
$$\Delta V_2 = (2.67 \text{ A})(2.00 \Omega) = 5.33 \text{ V},$$

$$\Delta V_4 = (2.67 \text{ A})(4.00 \Omega) = 10.67 \text{ V}$$

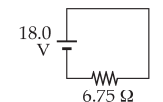
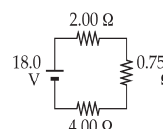
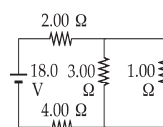
$$\Delta V_p = 18.0 \text{ V} - \Delta V_2 - \Delta V_4 = 2.00 \text{ V} (= \Delta V_3 = \Delta V_1)$$

$$P_3 = \frac{(\Delta V_3)^2}{R_3} = \frac{(2.00 \text{ V})^2}{3.00 \Omega} = \boxed{1.33 \text{ W}} \text{ in } 3.00 \Omega$$

$$P_1 = \frac{(\Delta V_1)^2}{R_1} = \frac{(2.00 \text{ V})^2}{1.00 \Omega} = \boxed{4.00 \text{ W}} \text{ in } 1.00 \Omega$$



CIRCUIT REDUCTION



7. Using Kirchoff's rules,
 (a) find the current in each resistor on the diagram (right)
 (b) Find the potential difference between points *c* and *f*.
 Which point is at the higher potential?

SOLUTION: we name the currents as shown

$$[1] \quad 70.0 - 60.0 - I_2(3.00 \text{ k}\Omega) - I_1(2.00 \text{ k}\Omega) = 0$$

$$[2] \quad 80.0 - I_3(4.00 \text{ k}\Omega) - 60.0 - I_2(3.00 \text{ k}\Omega) = 0$$

$$[3] \quad I_2 = I_1 + I_3$$

(a) Substituting for I_2 and solving the resulting simultaneous equations yields

$$I_1 = \boxed{0.385 \text{ mA}} \text{ (through } R_1)$$

$$I_3 = \boxed{2.69 \text{ mA}} \text{ (through } R_3)$$

$$I_2 = \boxed{3.08 \text{ mA}} \text{ (through } R_2)$$

(b) $\Delta V_{cf} = -60.0 \text{ V} - (3.08 \text{ mA})(3.00 \text{ k}\Omega) = \boxed{-69.2 \text{ V}}$

