

## Week # 9                      MATH1004

### 7.3 Integration by Parts

- Reverse Operation According to Product Rule

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$$

By integrating both sides and rearranging terms

$$\int u \frac{dv}{dx} dx = u(x)v(x) - \int v \frac{du}{dx} dx + C$$

which is commonly written as

$$\int u dv = uv - \int v du$$

Example: Evaluate  $\int x \sin x dx$

- **The Table Method**

This is a more rapid way of setting up the Integration by Part environment.

$$\int u \frac{dv}{dx} dx$$

The following special table is considered

$u(x)$	$v'(x)$
Derivatives of $u(x)$	Integrals of $v'(x)$
$\downarrow$	$\downarrow$

*When do we stop calculating the entries in the Table?*

Look at every row of the table and see if any of the products of the two quantities in the row can be integrated without much effort. If you find such a row or see zero in the left column, stop filling in the table.

Let's say you decide to stop at row "n". Then the last term in your answer must be the integral of the **product** of the two functions in that row by the constant  $(-1)^{n-1}$ .

For example, you want to stop in row 4:

$$\int u \frac{dv}{dx} dx = u(x)v(x) - u'(x)v_{(1)}(x) + u''(x)v_{(2)}(x) - u'''(x)v_{(3)}(x) + \int u^{(4)}(x)v_{(3)}(x) dx$$

**Example:** Evaluate:

$$\int x \sin^{-1}\left(\frac{1}{x}\right) dx$$

- **7.3.1 The Product of a Polynomial and a Sine or Cosine**

$$\int (\text{polynomial in } x)(\text{sine/cosine function in } x)dx$$

Use table method by considering polynomial in the first column and Sine/Cosine function in the second column and stop when you see a zero in the left column.

**Example:** Evaluate

$$\int 2x^3 \cos 2x dx, \quad \int (x^3 + 2x^2 - x + 3) \sin(3x + 4) dx$$

- **7.3.2 The Product of Polynomial and Exponential**

$$\int (\text{polynomial in } x)(\text{exponential function in } x)dx$$

Place the polynomial on the left column of table and differentiate it until get the zero, while on the right keep integrating exponential function.

**Example:** Evaluate

$$\int x e^x dx \quad \int x^2 e^{3x} dx$$

- **7.3.3 The Product of Polynomial and Logarithm**

$$\int (\text{polynomial in } x)(\text{logarithm function in } x)dx$$

Integrate the polynomial and differentiate the logarithm and stop after one step.

In the case that the problem involves the power of logarithm, you need to use substitution method to convert the logarithm to exponential and use table method for polynomial and exponential.

**Examples:** Evaluate

$$\int \ln x dx, \quad \int x^2 \ln x dx, \quad \int x^4 (\ln x)^3 dx$$

- **7.3.4 The Product of Exponential and Sine or Cosine**

Exponential in the left side and Sine/Cosine in the right side of table. In this case we note that the functions appearing in the third row are the same as the first, so we stop at the third row.

**Example:** Evaluate

$$I = \int e^{2x} \sin 3x dx$$

- **MY CAR method**

We can simplify the above method by using **MY CAR**; “**MULTIPLY**”, “**CHANGE SIGN**”, “**ADD 1**”, “**RECIPROCAL**”

**Example:**

1. Evaluate

$$I = \int \sin 3x \cos 4x dx, \quad I = \int \sin(\ln x) dx$$

2. Evaluate

$$\int x^2 \sin x dx, \quad \int x^4 \ln x dx, \quad \int \sin 2x \cos 4x dx$$