

8.3 The Volume of a Solid of Revolution

- The volume of a right circular cylinder is

$$\text{Volume of Cylinder} = \pi(\text{radius})^2(\text{height}) = \pi r^2 h.$$

- We can derive the volume of a cylinder with a cylindrical hole, and hole is a cylinder in its own. V_{hole} , of the remaining solid which is a shell, is given by

$$V_{\text{hole}} = 2\pi(\text{average radius})(\text{width of wall})(\text{height}) \quad (1)$$

where

$$\text{average radius} = \frac{r_{\text{out}} + r_{\text{in}}}{2} \quad \text{wall width} = r_{\text{out}} - r_{\text{in}}$$

Examples:

1. Find the volume of the cylindrical solid (of revolution) obtained by rotating the vertical line segment whose ends are at (x, x^2) and $(x, 1)$, and whose width is defined by the symbol dx , about the y -axis.

2. Find the volume of the cylindrical solid of revolution obtained by rotating the horizontal line segment whose ends are at $(0, y)$ and (\sqrt{y}, y) and whose height is defined by the symbol dy , about the y -axis, $0 < y < 1$.

3. Find the volume of the cylindrical solid of revolution obtained by rotating the vertical line segment whose ends are at $(x, 0)$ and $(x, 2x - x^2)$ and whose width is defined by the symbol dx , about the y -axis. $0 < x < 2$

4. Find the volume of the cylindrical solid of revolution obtained by rotating the vertical line segment whose ends are at $(x, 0)$ and $(x, 2x - x^2)$ and whose width is defined by the symbol dx , about the x -axis.

• Finding the Volume of a Solid of Revolution

1. **Sketch the region**

2. **Decide on a typical slice** The rule of thumb here is just lie the one for areas in the previous section: For rotation about the either the x -axis or y -axis.

i If it easier to describe the region with function of x , use a vertical slice, otherwise

ii Write all the expressions as functions of y (by finding the inverse functions) and use a horizontal slice.

3. **Find the volume of the slice, dV** Use (1).
4. **Find the limits of integration** These are obtained by finding the *extremities* of the region.
5. **Write down the definite integral for the volume** drop all terms containing the square of either dx or dy from the expression for dV .
6. **Evaluate the definite integral**

Examples:

1. Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2$, the y -axis and the line $y = 1$ about the y -axis.
2. Find the volume of the cylindrical solid of revolution obtained by rotating the region bounded by the curves $y = 2x - x^2$ and $y = 0$ about y -axis.
3. Find the volume of the cylindrical solid of revolution obtained by rotating the region bounded by the curves $y = 2x - x^2$ and $y = 0$ about x -axis.