

- **Function**

A function f is a rule which associates with each object (say x , which also called *independent variable*) from one set named the *domain*, a **single** object (say $f(x)$, which also called *dependent variable*) from a second set called the *range*.

$Dom(f)$ is domain of f

$Ran(f)$ is range of f

x is just a symbol and could be anything else like \square , \diamond , \heartsuit .

- **Box Method:**

If $f(x) = x^2 + 1$, it could be write as $f(\square) = \square^2 + 1$ where \square could be any statement, eg. $3x + 2$, so

$$\begin{aligned} f(\boxed{3x+2}) &= \boxed{3x+2}^2 + 1 \\ &= 9x^2 + 4 + 12x + 1 \\ &= 9x^2 + 12x + 5 \end{aligned}$$

- **Examples:**

$$g(x) = x^2 \rightarrow g(3x + 4) = ?$$

$$f(x) = x^2 + 2x - 1 \rightarrow f(-1) = ?; \quad f\left(\frac{1}{2}\right) = ?$$

$$g(t) = t^3 \sin t \rightarrow g(x + 1) = ?$$

$$f(x) = 2x^2 - x + 1 \rightarrow \frac{f(x+h) - f(x)}{h} = ?$$

Let f be defined by

$$f(x) = \begin{cases} x + 3, & \text{if } -1 \leq x \leq 2 \\ x^2, & \text{if } x > 2 \end{cases}$$

This function is said to be **defined in pieces**, since it takes on different values depending on values of x

- **Examples:**

$$f(x) = \begin{cases} x+1 & -1 \leq x \leq 0 \\ -x+1 & 0 < x \leq 2 \\ x^2 & 2 < x \leq 6 \end{cases} \rightarrow f(0)=?; f(3)=?; f(3x+2)=? \text{ if } 0 < x < 1$$

$$f(x) = \begin{cases} x-1 & 0 \leq x \leq 2 \\ 2x & 2 < x \leq 4 \end{cases} \rightarrow f(x+1)=? \text{ if } 1 < x \leq 2$$

- **Absolute value of a function**

The function whose rule is defined by setting

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

is called the **absolute value** function.

In general

$$|\square| = \begin{cases} \square & \text{if } \square \geq 0 \\ -\square & \text{if } \square < 0 \end{cases}$$

Note that:

$$|x| \leq y \iff -y \leq x \leq y, \quad |x| \geq y \iff x \geq y \text{ or } x \leq -y$$

In general

$$|\square| \leq \triangle \iff -\triangle \leq \square \leq \triangle, \quad |\square| \geq \triangle \iff \square \geq \triangle \text{ or } \square \leq -\triangle$$

Steps in removing absolute value in a function f :

1. Look at that part of f with the absolute values.
2. Put all the stuff between the vertical bars in a box.
3. Use the definition of the absolute value.
4. Remove the boxes and replace them by parentheses.
5. Solve the inequalities involving x 's for the symbol x .
6. rewrite f in pieces.

- **Example** Remove the absolute value for the following functions.

$$f(x) = |1 - x^2|$$

$$f(x) = |3x + 4|$$

$$f(x) = x + |x| \quad -\infty < x < \infty$$

$$f(x) = |x^2 + 2x|$$

- **Important Trigonometric Identities:**

Identity: An equation which is true for any value of the variable for which the expressions are defined.

$$\sin(-x) = -\sin(x) \quad \text{odd}$$

$$\cos(-x) = \cos(x) \quad \text{even}$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

- **Inequality**

$$0 < x \leq y \rightarrow \frac{1}{x} \geq \frac{1}{y}, \quad x \leq y \rightarrow -x \geq -y$$

$$\left| \frac{x+1}{2x+3} \right| < 2$$

Triangle Inequality

$$|x + y| \leq |x| + |y| \quad |x - y| \geq \left| |x| - |y| \right|$$