

$$\underline{1.} \bullet \int_1^4 \frac{(1+\sqrt{x})}{\sqrt{x}} dx = \int 2u du = u^2 = (1+\sqrt{x})$$

$$u = 1 + \sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}} dx \quad \begin{aligned} &= (1+\sqrt{4})^2 - (1+\sqrt{1})^2 \\ &= 9 - 4 = 5 \end{aligned}$$

$$\bullet \int \cos^4 5x \sin 5x dx$$

$$u = \cos 5x \rightarrow du = -5 \sin 5x dx$$

$$= -\frac{1}{5} \int u^4 du = -\frac{1}{5} \frac{u^5}{5} = -\frac{1}{25} (\cos 5x)^5$$

$$\bullet \int_{-1}^1 \frac{x^2}{1+x^6} dx$$

$$u = x^3 \quad du = 3x^2 dx$$

$$= \int \frac{1}{3} \frac{1}{1+u^2} du = \frac{1}{3} \int \frac{1}{1+u^2} du = \frac{1}{3} \operatorname{Arctan}(u)$$

$$= \frac{1}{3} \operatorname{Arctan}(x^3) \Big|_{-1}^1 = \frac{1}{3} [\operatorname{Arctan}(1) - \operatorname{Arctan}(-1)]$$

$$= \frac{1}{3} \left( \frac{\pi}{4} - \left( -\frac{\pi}{4} \right) \right) = \frac{2\pi}{12} = \frac{\pi}{6}$$

(2)

$$\bullet \lim_{x \rightarrow \infty} \frac{d}{dx} \int_{\sqrt{3}}^{\sqrt{x}} \frac{r^3}{(r+1)(r-1)} dr$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x})^3}{(\sqrt{x}+1)(\sqrt{x}-1)} \times \frac{1}{2\sqrt{x}} = 0$$

$$= \lim_{x \rightarrow \infty} \frac{x}{2(x-1)} = \frac{1}{2}$$

$$\bullet \int_0^1 x^3 dx \quad x^2 = u \quad 2x dx = du$$

$$= \int \frac{1}{2} 3^u du = \frac{1}{2} \frac{3^u}{\ln 3} = \frac{3^{x^2}}{2 \ln 3} \Big|_0^1$$

$$= \frac{3-1}{2 \ln 3} = \frac{1}{\ln 3}$$

$$\bullet \int \sqrt[3]{2-x} du = \int (2-x)^{\frac{1}{3}} dx$$

$$= - \frac{(2-x)^{\frac{4}{3}}}{\frac{4}{3}} = \frac{-3(2-x)^{\frac{4}{3}}}{4}$$

$$\bullet \int \frac{1+y}{\sqrt{1-y^2}} dy = \int \frac{1}{\sqrt{1-y^2}} dy + \int \frac{y}{\sqrt{1-y^2}} dy$$

$$= \text{Arcsin } y + \int \frac{1}{2} u^{-\frac{1}{2}} dy = \text{Arcsin } y + \frac{1}{2} \left( \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right)$$

$$1-y^2 = u \rightarrow -2y dy = du \quad = \text{Arcsin } y + \sqrt{1-y^2}$$

$$\int_0^{\frac{\sqrt{17}}{2}} x \sec(x^2) \tan(x^2) dx$$

$$u = x^2 \rightarrow du = 2x dx \rightarrow \frac{du}{2} = x dx$$

$$\int \sec(u) \tan(u) \frac{du}{2} = \frac{1}{2} [\sec u]$$

$$= \frac{1}{2} \sec(x^2) \Big|_0^{\frac{\sqrt{17}}{2}} = \frac{1}{2} [\sec(\frac{\pi}{4}) - \sec(0)]$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{2}$$