

Solution-Tutorial #3 - MATH1004

1. a) $\frac{d}{dx} \cos(xe^x) = -\sin(xe^x)(e^x + xe^x)$

b) $\frac{d}{dx} (x^2)^x \Big|_{x=1} = \frac{d}{dx} (e^{\ln x^{2x}}) \Big|_{x=1} = \frac{d}{dx} (e^{2x \ln x}) \Big|_{x=1}$
 $= e^{2x \ln x} (2 \ln x + 2) \Big|_{x=1} = 2$

c) $\frac{d}{dx} (\ln(x^3 \sin x^2)) = \frac{1}{x^3 \sin x^2} [3x^2 \sin x^2 + x^3 \cos x^2 (2x)]$

d) $\frac{d}{dx} (e^{-2x} \arctan x) = -2e^{-2x} \arctan x + \frac{e^{-2x}}{1+x^2}$

2. $(9x^2-1)(x+1) = (3x-1)(3x+1)(x+1) = 0$

Break Points: $\frac{1}{3}, -\frac{1}{3}, -1$

SDT

Interval	$3x-1$	$3x+1$	$x+1$	$P(x)$
$x < -1$	-	-	-	-
$-1 < x < -\frac{1}{3}$	-	-	+	+
$-\frac{1}{3} < x < \frac{1}{3}$	-	+	+	-
$x > \frac{1}{3}$	+	+	+	+

The Polynomial is Positive when

$-1 < x \leq -\frac{1}{3}$ OR $x \geq \frac{1}{3}$

3. $\frac{x^2-9}{x^2+x+1} = \frac{(x-3)(x+3)}{x^2+x+1}$

Break Points

$3, -3$

SDT

Interval	$x-3$	$x+3$	x^2+x+1	$r(x)$	The rational function is negative if $-3 < x < 3$ or $ x < 3$
$(-\infty, -3)$	-	-	+	+	
$(-3, 3)$	-	+	+	-	
$(3, \infty)$	+	+	+	+	

4. $P'(x) = 4x^3 - 18x^2 + 24x$

$$P''(x) = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2) = 0$$

$$12(x-1)(x-2) = 0 \quad \text{Break points: } 1, 2$$

Interval	$x-1$	$x-2$	$P''(x)$	$x < 1$ OR $x > 2$
$(-\infty, 1)$	-	-	+	
$(1, 2)$	+	-	-	
$(2, \infty)$	+	+	+	

5. $f(x) = \frac{1}{x^2-1}$ $f'(x) = \frac{-2x}{(x^2-1)^2} = 0$

$$f'(x) > 0 \quad \forall \quad x < 0 \quad \text{and } f \text{ is increasing}$$

6. $f(x) = x^3 - 3x + 2$ $f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$

$$\text{Critical Points } x = -1 \text{ and } x = 1$$