

Solution - Tutorial #2

MATH 10021

$$1. f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2+h-2}{h(\sqrt{2+h} + \sqrt{2})} = \frac{1}{2\sqrt{2}}$$

$$2. f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{(0+h)|0+h| - 0}{h}$$

$$= \lim_{h \rightarrow 0} |h| = 0 \quad \text{Since } \lim_{h \rightarrow 0^+} |h| = \lim_{h \rightarrow 0^-} |h| = 0$$

$$3. y'(1) = \frac{(2x-1)\sqrt{x} - \frac{1}{2\sqrt{x}}(x^2+x+3)}{x} \bigg|_{x=1} = -\frac{1}{2}$$

$$4. \frac{dy}{du} = \frac{dy}{dt} \times \frac{dt}{du} = 3t^2 \left(\frac{1}{2\sqrt{u}} \right)$$

$$= 3(\sqrt{u} + 6)^2 \times \frac{1}{2\sqrt{u}} \bigg|_{u=9} = \frac{81}{2}$$

$$5. \quad x^2 + 2x + y^2 - 4y - 24 = 0$$

$$2x + 2 + 2y y' - 4y' = 0$$

$$y'(2y - 4) = -x - 2 \quad y' = \frac{-2x - 2}{2y - 4} \quad |_{(4,0)}$$

$$m = y' \big|_{(4,0)} = \frac{-8 - 2}{-4} = +2.5$$

$$\text{Tangent line: } y - 0 = 2.5(x - 4)$$

$$y = 2.5(x - 4)$$

$$6. \quad \frac{d}{dx} \left(\frac{2x+3}{\sin x^2} \right) = \frac{2 \sin x^2 - 2x \cos x^2 (2x+3)}{(\sin x^2)^2}$$

$$7. \quad \frac{d}{dx} g^{-1}(x) = \frac{1}{g'(g^{-1}(x))}$$

$$y = x^2 + x \rightarrow x^2 + x - y = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+4y}}{2}$$

$$\text{sin } y \geq -\frac{1}{2} \quad x = \frac{-1 + \sqrt{1+4y}}{2}$$

$$\text{so } g^{-1}(x) = \frac{-1 + \sqrt{1+4x}}{2} \quad g'(x) = 2x + 1$$

$$\frac{d}{dx} g^{-1}(x) = \frac{1}{2 \left(\frac{-1 + \sqrt{1+4x}}{2} \right) + 1} = \frac{1}{\sqrt{1+4x}}$$