

Solution For Tutorial #1

$$1- f(x) = \begin{cases} \frac{1}{x\sqrt{x^2-1}} & x > 1 \\ \frac{-1}{x\sqrt{x^2-1}} & x < -1 \end{cases}$$

$$2- \frac{3}{2x-1} \leq 0 \rightarrow 2x-1 < 0 \rightarrow x < \frac{1}{2}$$

$$x^2 - 9 \leq 0 \rightarrow x^2 \leq 9 \rightarrow |x| \leq 3$$

$$\rightarrow -3 \leq x \leq 3$$

$$3- \lim_{x \rightarrow \pi^+} \frac{\cos x}{x - \pi} = -\infty \quad \text{since} \quad \lim_{x \rightarrow \pi^+} \cos x = -1$$

$$\lim_{x \rightarrow \pi^+} x - \pi = 0$$

$$\bullet \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^2} = 0 \quad \text{since}$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^2} = \lim_{x \rightarrow 0} \frac{\tan x (1 - \cos x)}{x^2} =$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{1}{\cos x} \right) \left(\frac{1 - \cos x}{x} \right) = 1 \times 1 \times 0 = 0$$

$$\bullet \lim_{x \rightarrow 0} \frac{\cos(2x)}{|x|} = \infty \quad \text{Since}$$

$$\cos 2x \rightarrow 1$$

$$x \rightarrow 0$$

$$|x| \rightarrow 0$$

$$x \rightarrow 0$$

$$\bullet \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} |x-1| = 0$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{\sin(x-1)}{x-1} = 1$$

So $\lim_{x \rightarrow 1} f(x)$ does not exist

$$\bullet \lim_{x \rightarrow \infty} \sqrt{x^2+1} - x = 0 \quad \text{Since}$$

$$\lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+1} - x)(\sqrt{x^2+1} + x)}{\sqrt{x^2+1} + x} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x^2+1} + x}$$

$$x \rightarrow \infty$$

$$f(x) = \begin{cases} \frac{|x|}{x} - 1 & x \neq 0 \\ 1 & x = 0 \end{cases}$$

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$$= \begin{cases} 0 & x > 0 \\ 1 & x = 0 \\ -2 & x < 0 \end{cases} \quad \text{check continuity at } x = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = 0 \quad \lim_{x \rightarrow 0^-} f(x) = -2 \quad \text{So}$$

$x = 0$ is discontinuity point

$$f(x) = \begin{cases} x^4 - 1 & x \neq 0 \\ -0.99 & x = 0 \end{cases} \quad \begin{aligned} \lim_{x \rightarrow 0} f(x) &= -1 \\ f(0) &= -0.99 \end{aligned}$$

So $f(x)$ is discontinuous at $x = 0$

$$f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x^3 - 1} & x \neq 1 \\ -\frac{1}{3} & x = 1 \end{cases} \quad \text{is continuous function since}$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^3 - 1} = \lim_{x \rightarrow 1} \frac{(x-2)(x+1)}{(x-1)(x^2+1+x)} = \lim_{x \rightarrow 1} \frac{x-2}{x^2+1+x} = -\frac{1}{3}$$

$$x \rightarrow 1$$

$$f(1) = -\frac{1}{3}$$