

MATH1004F — Solution-Test 5 — 7:35–8:25, Nov. 19 2014

Total: 20 marks

Multiple Choice (No Partial Mark), circle the best possible answer

1. [2 points] Evaluate the indefinite integral $\int (x-1)e^{-x}dx$.

- (a) The integral diverges (b) $xe^{-x} + C$ (c) $-xe^{-x} + C$ (**) (d) $(x-1)e^x + C$

2. [2 points] Find

$$\int_0^{\frac{\pi}{2}} x \cos 2x dx$$

- (a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $\frac{-1}{2}$ (**)

3. [2 points] Evaluate the finite integral

$$\int_0^1 x^2 e^{0.5x} dx$$

- (a) $10e^{0.5} - 16$ (**) (b) $e^{0.5} - 8$ (c) $e^{0.5} - 6$ (d) $2e^2$

4. [2 points] Evaluate

$$\int_0^1 x^4 \ln x dx$$

- (a) $\frac{1}{25}$ (b) $\frac{-1}{5}$ (c) $-\frac{1}{25}$ (**) (d) $\frac{1}{5}$

5. [2 points] Evaluate

$$\int t^2 \ln 2t dt$$

- (a) $\frac{1}{3}t^3 \ln(2t) - \frac{t^3}{9} + C$ (**) (b) $\frac{1}{3}t^2 \ln t - \frac{t}{9} + C$

- (c) $3(\ln 2t)^2 + \frac{1}{9} + C$ (d) $2t^2(\ln 2t) + \frac{t}{9} + C$

Long Answer Questions, you have to show your steps.

5. [3+4+3 points] Evaluate the following integrals using any method:

a.

$$I = \int_0^{\frac{\pi}{6}} e^{2x} \sin 3x dx$$

Solution:

Using integration by part

e^{2x}	$\sin 3x$
$2e^{2x}$	$-1/3 \cos 3x$
$4e^{2x}$	$-1/9 \sin 3x$

so,

$$\int e^{2x} \sin 3x dx = \square(-1/3 e^{2x} \cos 3x + 2/9 e^{2x} \sin 3x)$$

by using MY CAR method $\square = 9/13$, so

$$I = \frac{9}{13}((-1/3 e^{\pi/3} \cos(\pi/2) + 2/9 e^{\pi/3} \sin(\pi/2) + 1/3) = \frac{9}{13}(2/9 e^{\pi/3} + 1/3)$$

b.

$$I = \int \cos(\ln x) dx$$

Solution:

First consider $\ln x = u \rightarrow x = e^u \rightarrow dx = e^u du$, so

$$\int \cos(\ln x) dx = \int \cos u e^u du$$

by using integration by part

e^u	$\cos u$
e^u	$+\sin u$
e^u	$-\cos u$

so

$$\int \cos u e^u du = \square(e^u \sin u + e^u \cos u)$$

where by using MY CAR method $\square = 1/2$, finally

$$\int \cos(\ln x) dx = 1/2(x \sin(\ln x) + x \cos(\ln x)) + C$$

c.

$$I = \int x^3 (\ln x)^3 dx$$

Solution: By substitution

$$\ln x = t \rightarrow x = e^t \rightarrow dx = e^t dt$$

then $\int x^3 (\ln x)^3 dx = \int e^{4t} t^3 dt$, now by using integration by part

t^3	e^{4t}
$3t^2$	$1/4e^{4t}$
$6t$	$(\frac{1}{4})^2e^{4t}$
6	$(\frac{1}{4})^3e^{4t}$
0	$(\frac{1}{4})^4e^{4t}t$

$$\begin{aligned}\int e^{4t}t^3dt &= (\frac{1}{4})t^3e^{4t} - 3(\frac{1}{4})^2t^2e^{4t} + 6(\frac{1}{4})^3te^{4t} - 6(\frac{1}{4})^4e^{4t} + C \\ &= x^4(((\frac{1}{4})\ln x)^3 - 3(\frac{1}{4})^2\ln x^2 + 6(\frac{1}{4})^3\ln x - 6(\frac{1}{4})^4) + C\end{aligned}$$