

MATH1004F —Solution-Test 4 — 19:35–20:25, Nov. 12 2014

Multiple Choice (No Partial Mark), circle the best possible answer

1. [2 points] An antiderivative of $f(x) = \cot(4x + 5)$ is given by

(a) $4 \ln(\sec(4x + 5)) - 1$ (b) $\frac{1}{4} \ln |\cos(4x + 5)| + 2$

(c) $\frac{1}{4} \ln |\sin(4x + 5)| + 3$ (**) (d) $\frac{1}{4} \ln |\csc^2(4x + 5)|$

2. [2 points] Find the $\sum_{i=1}^n 2\left(\frac{i}{n}\right)^2$

(a) $\frac{6}{n^2}$ (b) $\frac{(n+1)(2n+1)}{3n}$ (**) (c) $\frac{n(n+1)(2n+1)}{3}$ (d) $\frac{n(n+1)(n+1)^2}{6}$

3. [2 points] Evaluate

$$\int_{-1}^1 (3x^2 - 2x + 1) dx$$

(a) 5 (b) 0 (c) 4 (**) (d) 6

4. [2 points] Evaluate

$$\int_0^1 5^{-x} dx$$

(a) $\frac{1}{\ln 5}$ (b) $\frac{4}{5 \ln 5}$ (**) (c) $\frac{4}{\ln 5}$ (d) $\frac{1}{\ln 25}$

5. [2 points] Find the value of $f(x) = \int 4xe^{x^2} dx$ where $f(0) = 1$

(a) $4e^{x^2} - 1$ (b) $2e^{x^2} - 1$ (**) (c) $2e^x + 1$ (d) $4e^x - 1$

Long Answer Questions, you have to show your steps.

6. [3 points] Given that

$$I(x) = \int_x^{2x^3-1} \frac{1}{1 + \sqrt{1-t^2}} dt$$

find $I'(x)$ using any method.

Sol:

$$\frac{d}{dx} \int_x^{2x^3-1} \frac{1}{1 + \sqrt{1-t^2}} dt = \frac{1}{1 + \sqrt{1-(2x^3-1)^2}} \times 6x^2 - \frac{1}{1 + \sqrt{1-x^2}}$$

7. [4 points] Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{x^2}{y^3}$$

subject to the initial condition $y(0) = 1$.

Sol:

$$y^3 dy = x^2 dx \rightarrow \int y^3 dy = \int x^2 dx \rightarrow \frac{y^4}{4} = \frac{x^3}{3} + C \rightarrow \frac{1}{4} = C \text{ then } \frac{y^4}{4} = \frac{x^3}{3} + \frac{1}{4}$$

8. [3 points] Evaluate

$$\int x 5^{2x^2-3} dx$$

Sol:

$$\frac{1}{4 \ln 5} \int 4x 5^{2x^2-3} \ln 5 dx = \frac{1}{4 \ln 5} [5^{2x^2-3}]$$