

MATH1004F — Solution-Test 2 — 7:35–8:25, Oct. 08 2014

Total: 20 marks

Multiple Choice (No Partial Mark), circle the best possible answer

1. [2 points 1+1] Evaluate

(i) $\tan(\sin^{-1} \frac{\sqrt{3}}{2}) :$ (a) $1/3$ (b) $\sqrt{3}/3$ (c) $\sqrt{3}$ (**) (d) 1

(ii) $\cos(\arcsin 0) :$ (a) 1 (**) (b) $1/2$ (c) ∞ (d) 0

2. [2 points] Let y be given implicitly as a differentiable function of x by $3x = 2xy + y^2$. Then the slope of the tangent line to the curve $y = y(x)$ at the point (x, y) where $x = 1, y = 1$ is equal to:

(a) 2 (b) $1/2$ (c) 4 (d) $1/4$ (**)

3. [2 points] Given that f is such that its inverse F exists, $f'(-7) = 3$, $F(4) = -7$, find the value of the derivative of F at $x = 4$.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (**) (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

4. [2 points] What is derivative of $\cos(x \sin x)$ at $x = \frac{\pi}{2}$

(a) 1 (b) -1 (**) (c) 0 (d) $\frac{\pi}{2}$

5. [2 points] Find $\lim_{x \rightarrow 0} \frac{\arctan x}{x^2}$

(a) ∞ (b) 0 (c) $-\infty$ (d) The limit does not exist. (**)

Long Answer Questions, you have to show your steps.

6. [3 points] Let $y = t^2 + \sin t$ and $t = \sqrt{u} + 2$. Find $\frac{dy}{du}$ when $u = 4$.

Sol:

$$\frac{dy}{du} = \frac{dy}{dt} \frac{dt}{du} = (2t + \cos t) \frac{1}{2\sqrt{u}} = \left[2(\sqrt{u} + 2) + \cos(\sqrt{u} + 2) \right] \frac{1}{2\sqrt{u}} \Big|_{u=4} = 2 + \frac{\cos 4}{4}$$

7. [2 points] Use the definition of the derivative to find derivative of $f(x) = |x - 2|$ at $x = 2$.

Sol:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{|h|}{h} = \begin{cases} \lim_{h \rightarrow 0^-} \frac{|h|}{h} = -1 \\ \lim_{h \rightarrow 0^+} \frac{|h|}{h} = 1 \end{cases}$$

So, derivative does not exist at $x = 2$

8. [2 points] What is derivative of $y = x^3 \arcsin(x^3)$

Sol:

$$y' = 3x^2 \arcsin x^3 + \frac{1}{\sqrt{1-x^6}}(3x^2 x^3) = 3x^2 \arcsin x^3 + \frac{3x^5}{\sqrt{1-x^6}}$$

9. [3 points] Find the tangent line to the curve $y = \frac{x^2 - x + 3}{\sqrt{x}}$ at $x = 1$.

Sol:

$y(1) = 3$, $y' = \frac{(2x-1)\sqrt{x} - (x^2-x+3)/2\sqrt{x}}{x} \rightarrow m = 1 - 3/2 = -1/2$, then the equation of tangent line is

$$y - 3 = -1/2(x - 1) \rightarrow y = -1/2x + 7/2$$