

# MATH1004F— Solution-Test 1 — 7:35–8:25, Sep. 24 2014

## Multiple Choice (No Partial Mark), circle the best possible answer

1. [2 points ] Let

$$f(x) = \begin{cases} 3c + x, & \text{if } x < 0; \\ 9 - x^2, & \text{if } x > 0. \end{cases}$$

If  $\lim_{x \rightarrow 0} f(x)$  does not exist, then

- (a)  $c = 3$               (b)  $c \neq 9$               (c)  $c \neq 0$               (d)  $c \neq 3$

Answer: (d)

2. [2 points] If  $f(x) = |x - 2| + 2$ , calculate  $L = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$

- (a)  $L=0$               (b)  $L=1$               (c)  $L=-1$               (d) The limit does not exist

Answer: (d)

3. [2 points] Evaluate  $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 9}$

- (a) The limit does not exist              (b) 0              (c)  $\frac{2}{3}$               (d)  $\frac{3}{2}$

Answer: (c)

4. [2 points ] Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}$

- (a)  $L=0$               (b)  $L=3$               (c)  $L=\infty$               (d) The limit does not exist

Answer: (b)

5. [2 points] Evaluate  $f(x - 1)$  if  $f(x) = x^2 + 2x + 1$

- (a)  $x^2 - 1$               (b)  $x^2$               (c)  $x^2 + 1$               (d) 2

Answer: (b)

## Long Answer Questions, you have to show your steps.

6. [2 points ] Remove the absolute value if  $f(x) = |3x - x^2|$ .

Sol:

$3x - x^2 = x(3 - x)$  which is positive if  $0 < x < 3$ , by using sign Table,so

$$f(x) = \begin{cases} 3x - x^2 & \text{if } 0 \leq x \leq 3 \\ x^2 - 3x & \text{if } x < 0 \text{ or } x > 3 \end{cases}$$

7. [3 points] Evaluate the  $\lim_{x \rightarrow 0} \frac{\sin(5x)}{\sin(2x)}$ .

Sol:

$$\lim_{x \rightarrow 0} \frac{5 \sin 5x / 5x}{2 \sin(2x) / 2x} = \frac{5}{2}$$

8. [3 points] Evaluate  $\lim_{x \rightarrow \infty} \sqrt{4x^2 + 3x - 5} - 2x$ .

Sol:

When  $x \rightarrow \infty$ , then the expression approaches to  $\infty - \infty$ , so need to simplify

$$\begin{aligned} \sqrt{4x^2 + 3x - 5} - 2x &= \frac{(\sqrt{4x^2 + 3x - 5} - 2x)(\sqrt{4x^2 + 3x - 5} + 2x)}{\sqrt{4x^2 + 3x - 5} + 2x} = \frac{4x^2 + 3x - 5 - 4x^2}{\sqrt{4x^2 + 3x - 5} + 2x} \\ &= \frac{3x - 5}{\sqrt{4x^2 + 3x - 5} + 2x} = \frac{3}{4} \end{aligned}$$

9. [2 points ] Determine the point of discontinuity of the following function

$$F(x) = \begin{cases} x^3 + 2 & \text{if } x < 0 \\ 2 - |x| & \text{if } 0 \leq x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$$

Sol:

$$\lim_{x \rightarrow 0^-} F(x) = 0 + 2 = 2, \quad \lim_{x \rightarrow 0^+} F(x) = 2 - 0 = 2, \quad F(0) = 2 \text{ then } \lim_{x \rightarrow 0} F(x) = F(0) = 2$$

$\lim_{x \rightarrow 1^-} F(x) = 2 - 1 = 1$ ,  $\lim_{x \rightarrow 1^+} F(x) = 2(1) = 2$ , then  $\lim_{x \rightarrow 1} F(x)$  does not exist and  $x = 1$  is discontinuity point.