MIDTERM 1

Microeconomic Theory III
ECO 3153A

Professor Rose Anne Devlin

September 24, 2014

Notes

There are 6 questions in this test and 50 points in total (plus a possible bonus of 4 marks). This means that you should budget about 1.6 minutes per mark.

Read over all of the questions. Do the questions that you know well first, then go to the ones that you are less sure of. I do not care in which order you answer the questions.

Basic calculators are allowed. No electronic device that can be programmed with text, or that has transmitting possibilities can be used.

No cellular phones are allowed.

1. Monotonic preferences imply that the indifference curve must be negatively sloped (true or false; marks only for explanation). (3 marks)

2. According to the Weak Axiom of Revealed Preference, if, with the price vector p^1, bundle x^1 is revealed preferred to bundle x^2, then it must be the case that if bundle x^2 is chosen at prices p^2, then: $\sum_{i=1}^{n} p_i^1 x_i^1 > \sum_{i=1}^{n} p_i^2 x_i^2$. (true or false; marks only for explanation). (3 marks)

3. Suppose a strictly quasi-concave utility function U(X), where $X = x_1, x_2, \ldots x_n$. Prove that the marginal rate of substitution between any two goods is invariant to any positive monotonic transformation of this function. (4 marks)

4. Suppose that an individual's preferences are described by the utility function $U(x_1, x_2) = x_1 x_2$.

   a) Write out the equation for an indifference curve. (2 marks)
   b) Calculate the Marshallian demand functions for goods 1 and 2. (4 marks)
   c) Suppose that the price of good 2 doubles, clearly show what will happen to the demand for good 1. (2 marks)
   d) Calculate his indirect utility function (IUF). (3 marks)
   e) Demonstrate that the IUF is homogeneous of degree zero in prices and income. (2 marks)

5. Suppose that an individual's preferences are described by the utility function $U(x_1, x_2) = x_1^{1/2} x_2^{1/2}$.

   a) Calculate her Hicksian demand functions. (4 marks)
   b) Calculate the expenditure (cost) function. (3 marks)
   c) Prove that Shephard's lemma holds. (3 marks)
   d) Suppose that the individual spends \$200 and $p_1 = 1$, $p_2 = 1$. What is the optimal bundle? (2 marks)
   e) Suppose that the price of good 1 doubles ($p_1 = 2$). Calculate her new Hicksian demand for goods 1 and 2. (4 marks)
   f) What is the level of income necessary so that she can purchase this new bundle at the new prices. (2 marks)

6. Suppose that preferences are described by: $U(x_1, x_2) = a x_1^{1/2} + x_2$

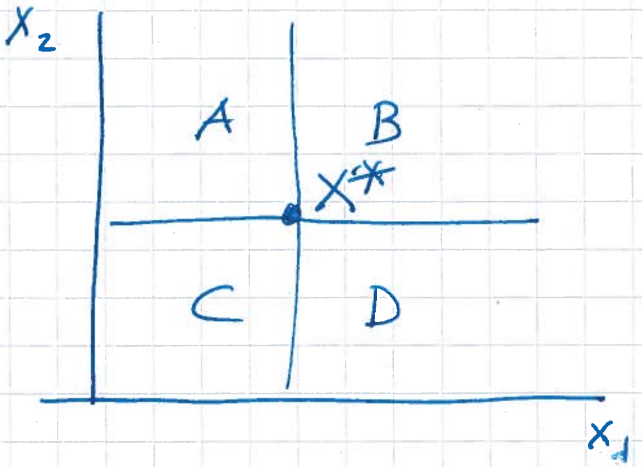
   a) Calculate the demand function for good 1 assuming both goods are purchased. (4 marks)
   b) What happens to the demand for good 1 when income increases? (2 marks)
   c) Calculate the demand for good 2 assuming both goods are purchased. (3 marks)

   BONUS part d)

   d) **Explain if it possible to have a corner solution for these preferences, and why. (4 marks)**

① Midterm #1 Eco 3153. 2014

1. Monotonic Preferences imply that more is better than less.



∴ if begin with bundle x^*, we know that all bundles in quadrant B are strictly preferred to x^*, and all bundles in quadrant C are strictly inferior to x^*. Thus all bundles for which the indiv is indifferent to x^* must lie in A + D. ⟹ Indiff. Curve is negative.

Any explanation which clearly defines monotonic preferences and the idea of trading off between good i and good j implying a negatively sloped IC. is required.

2. FALSE: if X^1 RP X^2 when prices are P^1 then if X^2 is chosen at P^2, it must be the case that X^1 is no longer affordable in order to satisfy the Weak Axiom of Revealed Pref.

i.e., $\sum_{i=1}^{n} P_i^2 X_i^1 > \sum_{i=1}^{n} P_i^2 X_i^2$

② Q3 With $u(x)$, $MRS_{i,j} = \frac{mu_j}{mu_i} = \frac{\partial u(x)/\partial x_j}{\partial u(x)/\partial x_i}$

$= u_j/u_i$

Let $T(u(x))$ be a positive, monotonic transformation.

Then. $MRS_{i,j} = \frac{\partial T(u(x))/\partial x_j}{\partial T(u(x))/\partial x_i}$

$= \frac{T' u_j}{T' u_i} = u_j/u_i$ = same as above.

(This question was on last year's midterm)

4. $u(x_1, x_2) = x_1 x_2$

a) $x_1 x_2 = \bar{u}$ $\quad x_2 = \frac{\bar{u}}{x_1}$ = equ'n I.C.

b) Calculate Marshallian demand fns.

$$\left.\begin{aligned} &\max_{x_1, x_2} x_1 x_2 \\ &\text{st } p_1 x_1 + p_2 x_2 = y \end{aligned}\right\} \mathcal{L} = x_1 x_2 + \lambda(y - p_1 x_1 - p_2 x_2)$$

$$\left.\begin{aligned} &① \ \partial\mathcal{L}/\partial x_1 : x_2 - p_1\lambda = 0 \\ &② \ \partial\mathcal{L}/\partial x_2 : x_1 - p_2\lambda = 0 \end{aligned}\right\} \Rightarrow \frac{x_2}{x_1} = \frac{p_1}{p_2} \quad (MRS = p_1/p_2)$$

③ $\partial\mathcal{L}/\partial\lambda : y - p_1 x_1 - p_2 x_2 = 0.$ $\quad \Rightarrow x_2 = \frac{p_1}{p_2} x_1$ — ④

put ④ into ③ $\quad y - p_1 x_1 - p_2\left[\frac{p_1 x_1}{p_2}\right] = 0 \quad y - 2p_1 x_1 = 0$

$$\boxed{x_1^* = \frac{y}{2p_1}}$$

(3) and $\boxed{X_2^* = \frac{P_1}{P_2}\left[\frac{y}{2P_1}\right] = \frac{y}{2P_2}}$

NB: If you used our "trick" $X_1 X_2 \Rightarrow$ transformed to $x_1^{1/2} x_2^{1/2} \Rightarrow X_1^* = \frac{y}{2P_1}$, $x_2^* = \frac{y}{2P_2}$

I will accept this. Also accept $MRS = \frac{P_1}{P_2}$ and budget constraint.

c) $X_1^* = \frac{y}{2P_1}$: If P_2 doubles it has <u>no impact</u> on x_1^*.

(you should think about what that implies about income & sub'n effects.....)

d) $IUF = X_1^* X_2^* = \left(\frac{y}{2P_1}\right)\left(\frac{y}{2P_2}\right) = \frac{y^2}{4P_1P_2}$

e) homo degree zero? change all p's and y by α

$$\frac{(\alpha y)^2}{4(\alpha P_1)(\alpha P_2)} = \frac{\cancel{\alpha^2} y^2}{\cancel{\alpha^2} 4P_1P_2} = \frac{y^2}{4P_1P_2} \cdot \underset{\substack{\uparrow \\ \text{degree } \phi.}}{\alpha^0}$$

$\Rightarrow$ If all p's and y change by same amnt, no impact on utility.

④ Q5: $u(x) = x_1^{1/2} x_2^{1/2}$

a) Hicksian: $\min_{x_1, x_2} p_1 x_1 + p_2 x_2$

st $x_1^{1/2} x_2^{1/2} = u$.

$$-\mathcal{L} = -p_1 x_1 - p_2 x_2 - \lambda (u - x_1^{1/2} x_2^{1/2})$$

$$\frac{\partial}{\partial x_1} \Rightarrow -p_1 + \lambda \tfrac{1}{2} x_1^{-1/2} x_2^{1/2} = 0 \quad (1)$$

$$\frac{\partial}{\partial x_2} \Rightarrow -p_2 + \lambda \tfrac{1}{2} x_1^{1/2} x_2^{-1/2} = 0 \quad (2)$$

$$\frac{\tfrac{1}{2} x_1^{-1/2} x_2^{1/2}}{\tfrac{1}{2} x_1^{1/2} x_2^{-1/2}} = \frac{p_1}{p_2}$$

$$\Rightarrow \frac{x_2}{x_1} = \frac{p_1}{p_2}$$

$$\Rightarrow x_2 = \frac{p_1}{p_2} x_1 \quad (4)$$

$$\frac{\partial}{\partial \lambda} = x_1^{1/2} x_2^{1/2} - u = 0 \quad (3)$$

put ④ into ③ $\quad x_1^{1/2} \left[\frac{p_1}{p_2} x_1 \right]^{1/2} - u = 0$

$$x_1 \left(\frac{p_1}{p_2} \right)^{1/2} = u \qquad x_1^* = u \left(\frac{p_2}{p_1} \right)^{1/2} = u \left(\frac{p_1}{p_2} \right)^{-1/2}$$

$$x_2^* = \frac{p_1}{p_2} \left[u \cdot \left(\frac{p_1}{p_2} \right)^{-1/2} \right] = \left(\frac{p_1}{p_2} \right)^{1/2} u.$$

b) Expenditure fn : $p_1 u \left(\frac{p_1}{p_2} \right)^{-1/2} + p_2 u \left(\frac{p_1}{p_2} \right)^{1/2} = c(p, u)$

$$c(p, u) = u \left(p_1 \left(\frac{p_1}{p_2} \right)^{-1/2} + p_2 \left(\frac{p_1}{p_2} \right)^{1/2} \right)$$

$$= u \left(p_1^{1/2} p_2^{1/2} + p_1^{1/2} p_2^{1/2} \right) = 2 u p_1^{1/2} p_2^{1/2}.$$

5 Q5 cont'd.

c) Shephard's Lemma: $\frac{\partial c}{\partial P_i} = x_i^*$ (Hicksian)

$$\frac{\partial c}{\partial P_1} = \frac{1}{2} 2 u p_1^{-1/2} p_2^{1/2} = u\left(\frac{P_2}{P_1}\right)^{1/2} \checkmark$$

$$\frac{\partial c}{\partial P_2} = \frac{1}{2} 2 p_1^{1/2} p_2^{-1/2} u = u\left(\frac{P_1}{P_2}\right)^{1/2} \checkmark$$

d) $y = 200, \quad P_1 = 1, \quad P_2 = 1$

We know that: $c(p, u) = 2 u p_1^{1/2} p_2^{1/2} = y.$

sub in $P_1 = 1, P_2 = 1$ $\quad 2u = y \quad$ if $y = 200$

$2u = 200 \quad \boxed{u = 100}$ this is the max level of utility that comes from spending \$200 efficiently.

$$\Rightarrow X_1^* = 100\left(\frac{1}{1}\right) = 100$$

$$X_2^* = 100$$

e) If P_1 goes to \$2.

$$X_1 = u\left(\frac{P_2}{P_1}\right)^{1/2} = 100\left(\frac{1}{2}\right)^{1/2}$$

$$= 100(0.7071) = 70.71$$

$$X_2 = 100\left(\frac{2}{1}\right)^{1/2} = 100(1.414)$$

$$= 141.42.$$

(6)

If $X_1 = 70.71$, $X_2 = 141.42$ } then u remains at 100. = new Hicksian demands.

f) Cost? $P_1 X_1 + P_2 X_2 = 2(70.71) + 141.42 = \$282.84$

∴ Income needs to increase by \$82.84 so that the individual can purchase ~~the~~ this new bundle.

Q6: $u(x_1, x_2) = a x_1^{1/2} + x_2$.

* THIS QUESTION WAS ASSIGNED (modified Q 4.12)

a) $\max_{x_1 x_2} u(x_1, x_2)$

$\text{st } P_1 x_1 + P_2 x_2 = y.$

$x_1 > 0$
$x_2 > 0$

$$\mathcal{L} = a x_1^{1/2} + x_2 + \lambda (y - P_1 x_1 - P_2 x_2)$$

1) $\dfrac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{2} a x_1^{-1/2} - \lambda P_1 = 0$

2) $\dfrac{\partial \mathcal{L}}{\partial x_2} = 1 - \lambda P_2 = 0$

3) $\dfrac{\partial \mathcal{L}}{\partial \lambda} = y - P_1 x_1 - P_2 x_2 = 0$

$$\frac{\frac{1}{2} a x_1^{-1/2}}{1} = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{a}{2 x_1^{1/2}} = \frac{P_1}{P_2}$$

$$\frac{2 x_1^{1/2}}{a} = \frac{P_2}{P_1}$$

$$2 x_1^{1/2} = \frac{a P_2}{P_1}$$

$$x_1 = \left(\frac{a P_2}{2 P_1}\right)^2$$

NOTE THAT THIS IS INDEPENDENT OF INCOME

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$\therefore\ x_1^* = \left(\frac{a P_2}{2P_1}\right)^2.$

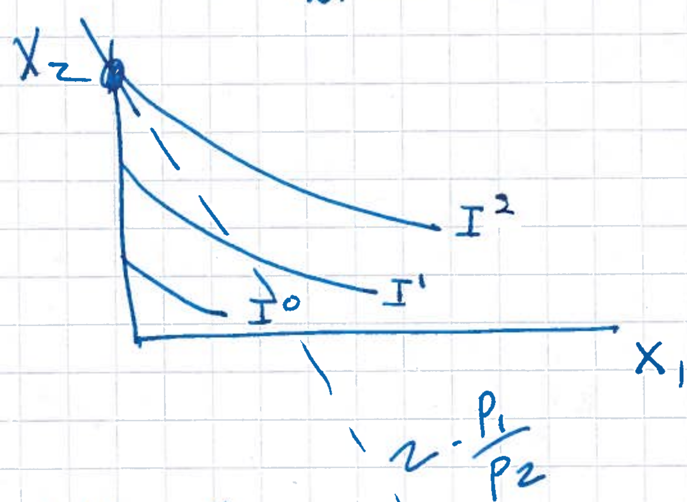
b) $\frac{\partial x_1^*}{\partial y} = 0$ NO INCOME EFFECT

c) sub x_1^* into (3)

$y - P_1\left[\frac{aP_2}{2P_1}\right]^2 - P_2 X_2 = 0 \quad P_2 X_2 = y - P_1\left(\frac{aP_2}{2P_1}\right)^2$

$$\boxed{\therefore\ X_2^* = \frac{y}{P_2} - \frac{P_2}{4P_1}a^2.}$$

BONUS. $X_1 = 0$ if $MRS_{2,1} < \frac{P_1}{P_2}$.



$\Rightarrow$ corner sol'n, choose only x_2..

note if $x_1 = 0$ still have utility. when the price of good 1 is high relative to its marginal utility, then $x_1 = 0$ and buy only x_2 (note: $MRS = \frac{MU_1}{1} \Rightarrow$ only a fn of MU_1). as MU_2 constant.