Question 1 - Transforming and combining random variables

a) A report from Center for Health Statistics says that the height of a 20-year-old man chosen at random is a random variable H with mean 5.8 feet and standard deviation 0.24 feet. Find the mean and standard deviation of the height J of a randomly selected 20-year-old man in inches. (Note: There are 12 inches in a foot) (4 points)

Answer:

$\mu_J = 12(\mu_H) = 12(5.8) = 69.6$ Inches (2 points)

$\sigma_J = 12(\sigma_H) = 12(0.24) = 2.88$ Inches (2 points)

b) Ms. Newell gave her class a 10-question multiple-choice quiz. Let X= the number of questions that a randomly selected student in the class answered correctly with a mean of 7.6 and a standard deviation of 1.32. To determine each student's grade on the quiz, Ms. Newell will multiply his or her number of correct answers by 10. Let G= the grade of a randomly chosen student in the class.

I) Find the mean of G. Show your work (2 points)

Answer: $\mu_G = 10(\mu_X) = 10(7.6) = 76$

II) Find the standard deviation of G. Show your work (2 points)

Answer: $\sigma_G = 12(\sigma_X) = 10(1.32) = 13.2$

III) How do the variance of G and the variance of X compare? (3 points)

Answer:

$$\sigma_G^2 = (13.2)^2 = 174.24$$
$$\sigma_X^2 = (1.32)^2 = 1.7424$$
$$\Rightarrow \sigma_G^2 = (10)^2 \sigma_X^2$$

c) David's firm is planning a major investment. The amount of profit (in millions of dollars) is uncertain, but an estimate gives the following probability distribution:

Profit	1	1.5	2	4	10
Probability	0.1	0.2	0.4	0.2	0.1

Based on this estimate $\mu_{X=3}$ and $\sigma_X = 2.52$

David owes its lender a fee of $200,000 plus 10% of the profits. So his firm actually retains Y=0.9X - 0.2 from the investment. Find the mean and standard deviation of Y. (4 points)

Answer:

$\mu_Y = 0.9\mu_X - 0.2 = 0.9(3) - 0.2 = 2.5$ Million (2 points)

$\sigma_Y = 0.9\sigma_X = 0.9(2.52) = 2.27$ Million (2 points)

Question 2 – Binomial probability calculations

Kobe Bryant, a professional basketball player in the NBA, has made 84% of his free throws during his career with the Los Angeles Lakers.

a) Calculate the probability that Bryant will make exactly three of his next five free throws. (3 points)

$$P(3,5) = \frac{5!}{(5-3)!3!}(0.84)^3(0.16)^{5-3}$$

$$P(3,5) = \frac{(5)(4)(3)(2)(1)}{(2)(1)(3)(2)(1)}(0.84)^3(0.16)^2$$

$$P(3,5) = (10)(0.5927)(0.0256) = 0.1517$$

b) Calculate the probability that Bryant will make less than five of his next six free throws. (3 points)

$$P(5,6) = \frac{6!}{(6-5)!5!}(0.84)^5(0.16)^{6-5}$$

$$P(5,6) = \frac{(6)(5)(4)(3)(2)(1)}{(1)(5)(4)(3)(2)(1)}(0.84)^5(0.16)^1$$

$$P(5,6) = (6)(0.4182)(0.16) = 0.4015$$

$$P(6,6) = \frac{6!}{(6-6)!6!}(0.84)^6(0.16)^{6-6}$$

$$P(6,6) = \frac{(6)(5)(4)(3)(2)(1)}{(1)(6)(5)(4)(3)(2)(1)}(0.84)^6(0.16)^0$$

$$P(6,6) = (1)(0.3513)(1) = 0.3513$$

$$P(x<5) = 1.0 - P(5,6) - P(6,6) = 1.0 - 0.4015 - 0.3513 = 0.2472$$

c) Calculate the probability that Bryant will make four or five of his next six free throws. (4 points)

$$P(4,6)=\frac{6!}{(6-4)!4!}(0.84)^4(0.16)^{6-4}$$
$$P(4,6)=\frac{(6)(5)(4)(3)(2)(1)}{(2)(1)(4)(3)(2)(1)}(0.84)^4(0.16)^2$$
$$P(4,6)=(15)(0.4979)(0.0256)=0.1912$$
$$P(5,6)=\frac{6!}{(6-5)!5!}(0.84)^5(0.16)^{6-5}$$
$$P(5,6)=\frac{(6)(5)(4)(3)(2)(1)}{(1)(5)(4)(3)(2)(1)}(0.84)^5(0.16)^1$$
$$P(5,6)=(6)(0.4182)(0.16)=0.4015$$
$$P(x=4 \text{ or } 5)=0.1912+0.4015=0.5927$$

Question 3 – Poisson probability calculations

The city of Vancouver Office of Emergency Management reported that 911 operators receive an average of 7.2 calls per hour. Assume that the number of 911 calls follows the Poisson distribution.

a) What is the probability that exactly five 911 calls will be made during the next hour? (3 points)

Answer

$$\lambda=7.2$$
$$P(5)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{(7.2^5)(2.71828^{-7.2})}{5!}$$
$$P(5)=\frac{(19{,}349.18)(0.000747)}{120}=0.1204$$

b) What is the probability that three or four 911 calls will be made during the next hour? (4 points)

$\lambda = 7.2$

$$P(3) = \frac{(7.2^3)(2.71828^{-7.2})}{3!} = \frac{(373.248)(0.000747)}{6} = 0.0465$$

$$P(4) = \frac{(7.2^4)(2.71828^{-7.2})}{4!} = \frac{(2,687.386)(0.000747)}{24} = 0.0836$$

$$P(3 \text{ or } 4) = 0.0465 + 0.0836 = 0.1301$$

c) What is the probability that three or more 911 calls will be made during the next hour? (5 points)

$\lambda = 7.2$

$$P(0) = \frac{(7.2^0)(2.71828^{-7.2})}{0!} = \frac{(1)(0.000747)}{1} = 0.0007$$

$$P(1) = \frac{(7.2^1)(2.71828^{-7.2})}{1!} = \frac{(7.2)(0.000747)}{1} = 0.0054$$

$$P(2) = \frac{(7.2^2)(2.71828^{-7.2})}{2!} = \frac{(51.84)(0.000747)}{2} = 0.0194$$

$$P(x \geq 3) = 1.0 - P(x < 3) = 1.0 - 0.0007 - 0.0054 - 0.0194 = 0.9745$$

d) What is the probability that exactly four 911 calls will be made during the next 30 minutes? (3 points)

$\lambda = 3.6$

$$P(4) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.6^4)(2.71828^{-3.6})}{4!}$$

$$P(4) = \frac{(167.9616)(0.027324)}{24} = 0.1912$$

Note: You should follow the plan-do-report format to present your solution.

Tim Hortons

Tim Hortons was founded in 1964 in Hamilton, Ontario, where it served coffee and doughnuts. As the chain has expanded throughout Canada, so have its product offerings, including the very popular Timbits, soups, cappuccinos, and breakfast sandwiches. In 1995 Tim Hortons merged with Wendy's International, which facilitated its expansion to the United States. Today there are over 2800 stores in Canada and over 400 in the United States. Although good taste and friendly service are important at Tim Hortons, fast service is valued, too, and several servers are often employed to serve customers at the same time.

Suppose you were the manager of a coffee shop with three servers, who each take an average of 1.8 minutes to serve a customer. You have, on average, a customer arriving every 0.8 minutes, and you're considering two options for ensuring fast service: (a) hiring a fourth server at an annual cost of $36,000 or (b) renting faster dispensing machines at an annual cost of $23,000, which would reduce the service time to 1.3 minutes, on average. You decide to base your decision on the number of customers who arrive during the time you can serve them. You don't want to have more than a 10% chance of more customers arriving than you can serve. For instance, with your current operation, you can serve three customers in 1.8 minutes, so you don't want the chance of more than three customers arriving in 1.8 minutes to be greater than 10%. What should you do—continue the current operation, hire a fourth server, or rent faster dispensing machines?

<u>Mini Case – Tim Hortons (20 Marks)</u>

PLAN (4 pts)	Setup: State the objective	What should you do:- continue the current operation, hire a fourth server, or rent faster dispensing machines?
DO (12 pts)	Mechanics: Customers arrive at random times independent of each other so the distribution of the number arriving in	Large format tables and graphs (if any) are placed below this PLAN/DO/REPORT table There are 3 options to analyse: (1) Current Operation (4pts) We can serve 3 customers in 1.8 minutes, so we need

	any period of time is Poisson.	to calculate the probability of more than 3 customers arriving in 1.8 minutes. The arrival rate is λ = 1.8/0.8 = 2.25 customers per 1.8 minutes. P(>3) = 1-P(0)-P(1)-P(2)-P(3) P(0) = 0.105399 P(1)= 0.237148 P(2)= 0.266792 P(3)= 0.200094 P(>3)= 0.190567 (2) Hire a fourth Server ($36,000) (4pts) We can serve 4 customers in 1.8 minutes, so we need to calculate the probability of more than 4 customers arriving in 1.8 minutes. The arrival rate is λ = 1.8/0.8 = 2.25 customers per 1.8 minutes. P(>4) = 1-P(0)-P(1)-P(2)-P(3)-P(4) P(0) = 0.105399 P(1)= 0.237148 P(2)= 0.266792 P(3)= 0.200094 P(4)= 0.1125 3 P(>4)= 0.078014 (3) Faster Dispensing Machines ($23,000) (4pts) We can serve 3 customers in 1.3 minutes, so we need to calculate the probability of more than 3 customers arriving in 1.3 minutes. The arrival rate is λ = 1.3/0.8 = 1.625 customers per 1.3 minutes.

		P(>3) = 1-P(0)-P(1)-P(2)-P(3) P(0) = 0.196912 P(1)= 0.319981 P(2)= 0.259985 P(3)= 0.140825 P(>3)= 0.082297
REPORT(4pts)	Conclusion: State the conclusion in the context of the original objective.	The current operation is unsatisfactory since the chance of customers arriving faster than they can be served is >10%. Either of the other options is fine since in each case the probability is reduced below 10%. We should rent faster dispensing machines since this is less costly than hiring a fourth server.