Overall Expectation
Identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions Solve problems invilving polynomial equations graphically and algebraically Demonstrate an understanding of solving polynomial inequalities.

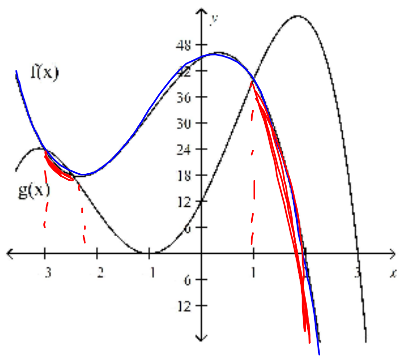
MHF 4U **Chapter 3&4 Test** **Name:** ____________________

1. For the function $f(x) = -2x^5 + 3x^3 - 1$ x^0 state:
   a) the end behaviour as $x \to \infty$ $f(x) \to$ $-\infty$
   b) symmetry (even, odd or neither) neither
   c) maximum possible number of turning points 4
   d) maximum possible number of zeros 5
2. What is the remainder when $x - 4$ is divided into $3x^4 - 9x^3 + 6x^2 - 8x + 11$? 267

   $f(4) =$

3. List all the test values you could reasonably try in the factor theorem to determine the factors of $f(x) = 2x^3 - 2x^2 + 3x - 3$. $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$
4. Solve: $-2x + 3 \leq -5$ $x \geq 4$

   $-2x \leq -8$
   $x \geq 4$



5. Estimate where $f(x) < g(x)$. $-3 < x < -2.5, x > 1$

6. Why can an even degree function have no x-intercepts while an odd degree function must have at least one?

odd degree have opp end beh. ∴ must pass through x-axis. Even degree have same end beh ∴ if lowest point is above axis and opens up no x-int.

7. Determine the cubic equation that has zeros at -4, -5 and 2, if $f(3) = -112$

$f(x) = a(x+4)(x+5)(x-2)$

$-112 = a(3+4)(3+5)(3-2)$

$a = -2$ $\therefore f(x) = -2(x+5)(x+4)(x-2)$

8. Perform long division and write **both** division statements for $(x^3 + 6x^2 - 11) \div (x^2 + x - 1)$.

$$
\begin{array}{r}
x + 5 \\
x^2 + x - 1 \overline{\smash{)}\, x^3 + 6x^2 + 0x - 11} \\
x^3 + x^2 - x \\
\hline
5x^2 + x - 11 \\
5x^2 + 5x - 5 \\
\hline
-4x - 6
\end{array}
$$

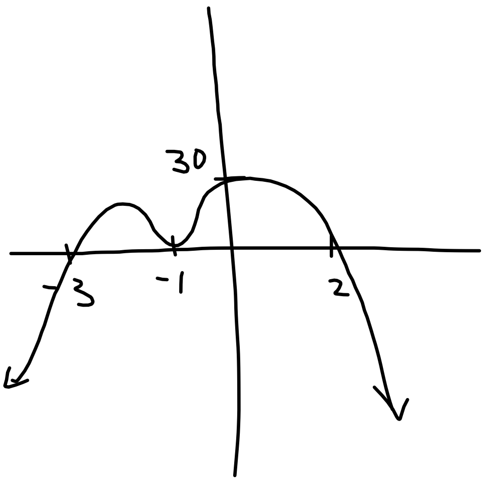
$x^3 + 6x^2 - 11 = (x+5)(x^2+x-1) + (-4x-6)$

$$\frac{x^3+6x^2-11}{x^2+x-1} = x + 5 + \frac{-4x-6}{x^2+x-1}$$

9. Sketch a graph for each following polynomial.

a) $f(x) = -5(x-2)(x+3)(x+1)^2$

$2 \quad -3 \quad -1^*$

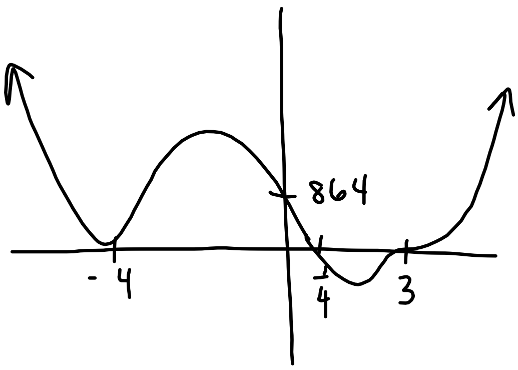


$-2 \quad x^2 \quad 4x \quad -x^3 \quad +$

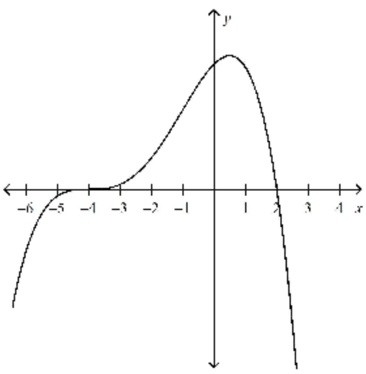
b) $f(x) = -2(x+4)^2(4x-1)(3-x)^3$

$-4^* \quad \frac{1}{4} \quad 3^*$

$f(0)=864$



10. Using the graph below, determine a possible equation for the function. Justify your answer.



flatte at -4

$f(x) = -5(x+4)^3(x-2)$

$a<0$

$\therefore$ as $x \to \infty$ $f(x) \to -\infty$

pass through on graph so exponent 1.

$f(2)=0$

11. The polynomial function $f(x) = ax^3 - 5x^2 - 12x + b$ has one of its zeros at $x = 2$, and when it is divided by $(x+1)$ the remainder is -3. Determine the values of a and b.

$f(-1)=-3$

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$-3 = -a - 5 + 12 + b$

$f(2)=0$

$0 = 8a - 20 - 24 + b$

② $-10 = -a + b$

① $44 = 8a + b$

$-10 = -a + b$

$54 = 9a$

$a = 6$

$-10 = -6 + b$

$b = -4$

12. Factor each of the following completely.

a) $x^4 + 4x^2 - 32$

$= (x^2 + 8)(x^2 - 4)$

$= (x^2 + 8)(x + 2)(x - 2)$

b) $2x^3 + 8x^2 + 3x + 12$

$= 2x^2(x+4) + 3(x+4)$

$= (x+4)(2x^2+3)$

c) $64x^6 - 1$

$(4x^2)^3 - (1)^3$

$(4x^2 - 1)(16x^4 + 4x^2 + 1)$

$(2x+1)(2x-1)(16x^4 + 4x^2 + 1)$

13. Solve each of the following.

a) $2x^3 + 9x^2 + 12x + 5 = 0$

$f(-1) = 0 \quad \therefore (x+1)$ is a factor

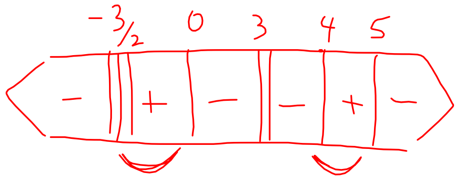
$0 = (x+1)(2x^2 + 7x + 5)$

$0 = (x+1)(2x+5)(x+1)$

$x = -1, -\frac{5}{2}$

b) $-2x(x-3)^2(x-4)(x-5)(2x+3)^3 \geq 0$

$0 \quad 3^* \quad 4 \quad 5 \quad -\frac{3}{2}^*$



$-\frac{3}{2} \leq x \leq 0 \quad 4 \leq x \leq 5, \; x = 3$

14. Solve the inequality both algebraically and graphically.

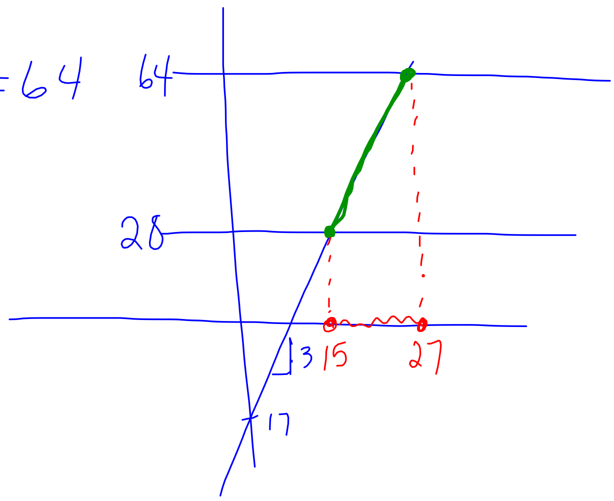
$28 \leq 2(4x+9) - 5(x+7) \leq 64$

$28 \leq 8x + 18 - 5x - 35 \leq 64$

$28 \leq 3x - 17 \leq 64$

$45 \leq 3x \leq 81$

$15 \leq x \leq 27$



15. Write an inequality that is never true (has no solution) and justify.

$x^4 + 4 \leq 0 \qquad (x-4)^2 + 3 < 3$

16. Determine the equation of a quartic function with single order zeros of $x = -2$ and $x = 4$. These are the only two x-intercepts. The functions y-intercept is -40, and it passes through the point (1, -27).

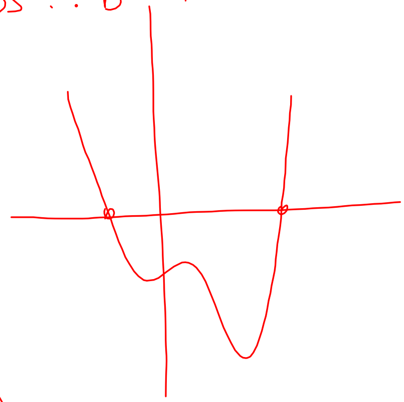
no zeros $\therefore b^2-4ac<0$

$f(x)=(x+2)(x-4)(ax^2+bx+c)$

$f(0)=-40$

$-40=2(-4)(c)$

$c=5$



$\therefore f(x)=(x+2)(x-4)(ax^2+bx+5)$

$f(1)=-27$

$-27=(3)(-3)(a+b+5)$

$-27=-9(a+b+5)$

$3=(a+b+5)$

$-2=a+b$

$1 \quad -3$

$b^2-4ac<0$

$(-3)^2-4(1)(5)$

$\therefore f(x)=(x+2)(x-4)(x^2-3x+5)$