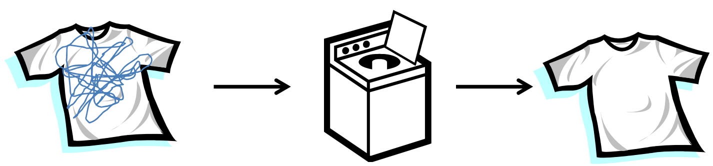
Idea: Function Notation

What is a Washing Machine? How does it work?



Washer(shirt) = clean shirt
Name_of_Function(Input) = Steps to take to get the output

Ex: $f(x) = x^2$

Name of this function: f
Input for this function: x
Steps to get the output: square x

Strategy: Box Method

Instead of writing a variable (like x) we could use a box symbol instead (□) to represent the input:

$$f(x) = x^2 + 2x + 3$$

$$f([\ \ ]) = [\ \ ]^2 + 2[\ \ ] + 3$$

This way we can have an easier time visualizing what we do with the input to get the output:

If the input is 3, this means 3 goes inside our box

$$f([3]) = [3]^2 + 2[3] + 3$$

If the input is h+2, this means h+2 goes inside our box

$$f([h+2]) = [h+2]^2 + 2[h+2] + 3$$

Example 1

Given $f(x) = 3x^2 + 2$, evaluate f(-2)

$$f(x) = 3x^2 + 2$$

$$f([\ \ ]) = 3[\ \ ]^2 + 2$$

$$f([-2]) = 3[-2]^2 + 2$$

$$f([-2]) = 3(4) \ + 2$$

$$f([-2]) = 12 + 2$$

$$f([-2]) = 14$$

$$\therefore f(-2) = 14$$

Example 2

Given $f(x) = 3x^2 + 2$, simplify $f(x+h) - f(x)$

$$f(x) = 3x^2 + 2$$

$$f([\ ]) = 3[\ ]^2 + 2$$

$$f([x+h]) = 3[x+h]^2 + 2$$

$$f([x+h]) = 3(x+h)(x+h) \ \ + 2$$

$$f([x+h]) = 3(x^2 + 2xh + h^2) + 2$$

$$f([x+h]) = 3x^2 + 6xh + 3h^2 + 2$$

$$\therefore f([x+h]) - f([x]) = 3x^2 + 6xh + 3h^2 + 2 - (3x^2 + 2)$$

$$f([x+h]) - f([x]) = 3x^2 + 6xh + 3h^2 + 2 - 3x^2 - 2$$

$$f([x+h]) - f([x]) = 6xh + 3h^2$$

Example 3

Given $f(x) = 3x^2 + 2$, and $g(x) = 4x + 1$, determine x when f(x) = g(x)

$$f(x) = g(x)$$

$$3x^2 + 2 = 4x + 1$$

$$3x^2 + 2 - 4x - 1 = 4x + 1 - 4x - 1$$

$$3x^2 - 4x + 1 = 0$$

To factor, we know the product is (3)(1) = 3 and the sum is (-4). This means we need 2 numbers that multiply to 3 and add to -4. These numbers are -3 and -1. This gives:

$$3x^2 - 3x - 1x + 1 = 0$$

$$3x(x-1) - 1(x-1) = 0$$

$$(3x-1)(x-1) = 0$$

$$\therefore 3x - 1 = 0$$

$$3x - 1 + 1 = 0 + 1$$

$$3x = 1$$

$$\frac{3x}{3} = \frac{1}{3}$$

$$x = \frac{1}{3}$$

$$\therefore x - 1 = 0$$

$$x - 1 + 1 = 0 + 1$$

$$x = 1$$

Composing Functions

When we place one function inside of another function, we call this composing functions. We use the following notation to show composition:

$$f(x) = x^2 + 2x + 3, g(x) = 2x + 1$$

$f \circ g(x) = f(g(x))$

$f \circ g(x) = [g(x)]^2 + 2[g(x)] + 3$

$f \circ g(x) = (2x+1)^2 + 2(2x+1) + 3$

$f \circ g(x) = 4x^2 + 4x + 1 + 4x + 2 + 3$

$f \circ g(x) = 4x^2 + 8x + 6$

$g \circ f(x) = g(f(x))$

$g \circ f(x) = 2[f(x)] + 1$

$g \circ f(x) = 2(x^2 + 2x + 3) + 1$

$g \circ f(x) = 2x^2 + 4x + 6 + 1$

$g \circ f(x) = 2x^2 + 4x + 7$

Note that the two compositions are not the same (can you think of a time when they would be the same?)