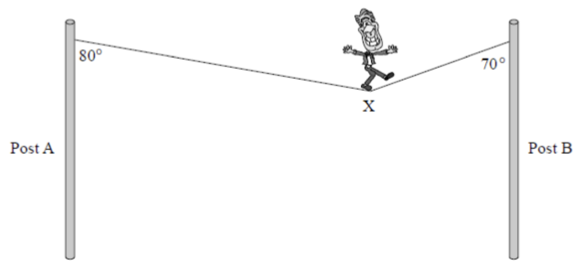
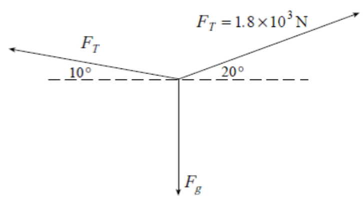
Practice Test: Equilibrium Name: ....

1) A circus performer walks across a wire stretched between two vertical posts. When the performer stands at position **X** as shown below, the tension in the short length of wire attached to post **B** is 1.8×10^3 N.



a) Draw and label a free body diagram showing the forces acting at position **X**. **(2 marks)**



← **2 marks**

b) What is the mass of the circus performer? **(5 marks)**

$\sum F_X = 0 \quad F_T \cos 10° = 1.8 \times 10^3 (\cos 20°)$ ← **2 marks**

$F_T = 1.72 \times 10^3$ N

$\sum F_y = 0 \quad 1.72 \times 10^3 \sin 10° + 1.8 \times 10^3 \sin 20° = F_g$

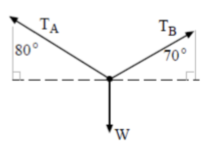
$F_g = 914$ N ← **2 marks**

$F_g = mg$

$m = \frac{914}{9.8} = 93$ kg ← **1 mark**

Alternate Response for Question 3.

a) Draw and label a free body diagram showing the forces acting at position **X**. **(2 marks)**

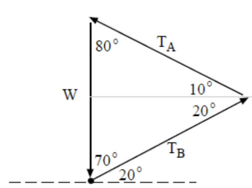


b) What is the mass of the circus performer? **(5 marks)**

In equilibrium

$0 = T_A + T_B + W$

$T_B = 1\,800$ N



3 marks

$\frac{\sin 30°}{W} = \frac{\sin 80°}{1\,800 \text{ N}}$ ←**1 mark**

$W = \left(\frac{\sin 30°}{\sin 80°}\right) 1\,800$ N

$W = \left(\frac{0.5}{0.98}\right) 1\,800$ N

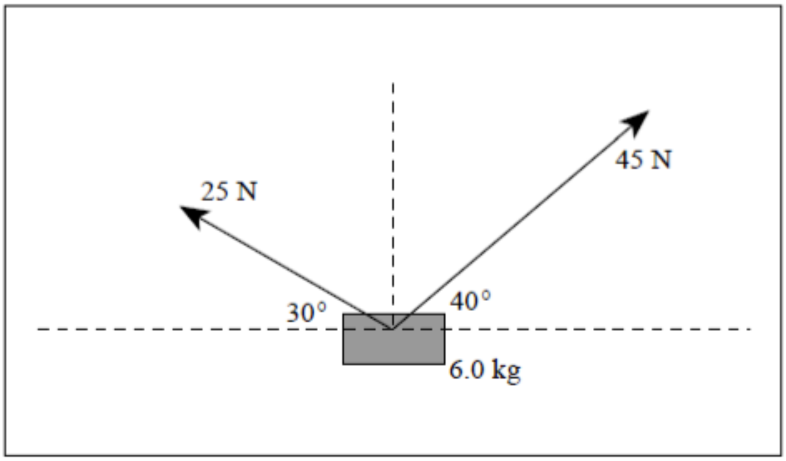
$W = 913.8$ N ← $\frac{1}{2}$ **mark**

$W = 914$ N

$M = \frac{914 \text{ N}}{9.8}$ ← $\frac{1}{2}$ **mark**

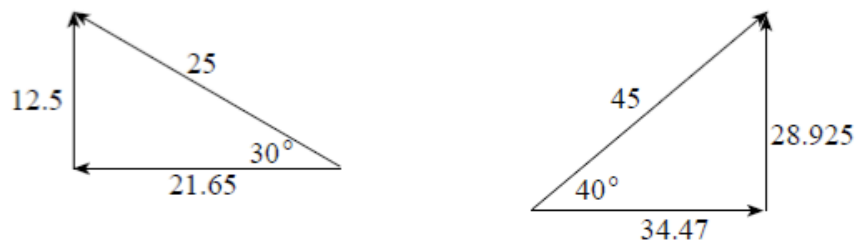
$M = 93$ kg

2) A 6.0 kg block is held at rest on a horizontal, frictionless air table. Two forces are pulling on this block in the directions shown in the diagram below.



What will be the magnitude of the acceleration on the 6.0 kg block at the moment it is released?

(7 marks)



Components:

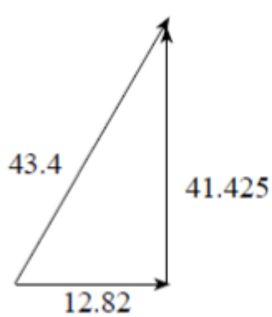
y direction

$F_y = (12.5 + 28.925) = 41.425\text{ N}$ ← **1½ marks**

x direction

$F_x = 34.47 - 21.65 = 12.82\text{ N}$ ← **1½ marks**

$F_{net} = 43.4\text{ N}$ **2 marks**

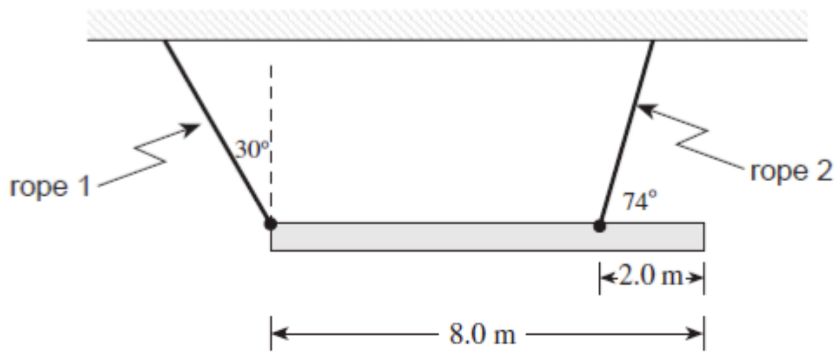


Answer:

$$a = \frac{F_{net}}{m} = \frac{43.4\text{ N}}{6.0\text{ kg}} = 7.2\text{ m/s}^2$$ **2 marks**

SEE ALTERNATE SOLUTION OVER:

3) The 8.0 m uniform beam shown below, suspended horizontally by two ropes, has a mass of 75 kg.



Determine the tension in rope 1 and the tension in rope 2. **(7 marks)**

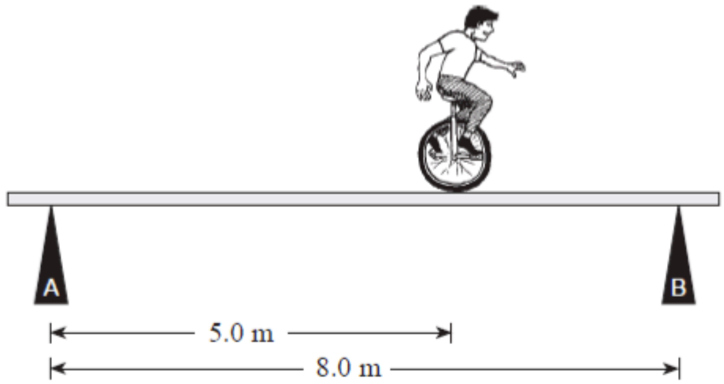
$$\Sigma\tau = 0 = -F_{T1}\cos 30°(6.0) + 75(9.8)(2.0)$$

$F_{T1} = 2.8\times 10^2\ \text{N}$ ← **4 marks**

$$\Sigma F_y = 0 = -75(9.8) + 283\cos 30° + F_{T2}\sin 74°$$

$F_{T2} = 5.1\times 10^2\ \text{N}$ ← **3 marks**

4) A circus performer on a unicycle of total mass 55 kg rides across a uniform 30 kg beam. The supports are placed equal distances from the ends of the beam.

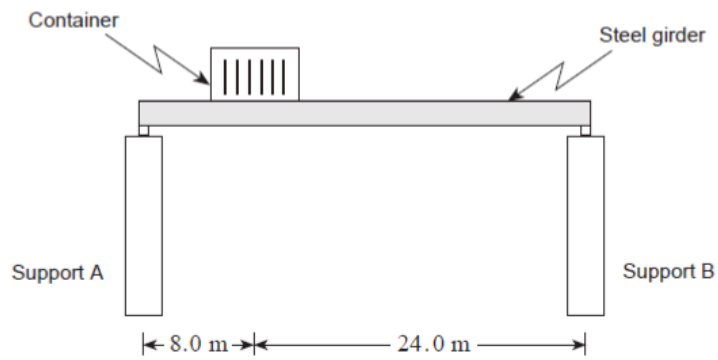


a) When he is at the position shown, determine the forces exerted by the supports on the beam. **(5 marks)**

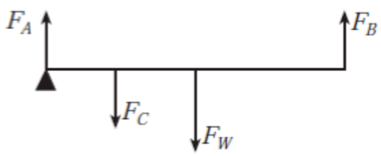
$$\Sigma\tau = 0$$
$$0 = -30(9.8)(4) + (-55)(9.8)(5) + F_B(8)$$
$$F_B = 480\text{ N}$$
← **3 marks**

$$\Sigma F = 0$$
$$0 = F_A + 480 - 30(9.8) - 55(9.8)$$
$$F_A = 350\text{ N}$$
← **2 marks**

5) A uniform 1 200 kg steel girder is supported horizontally at its endpoints as shown in the diagram.



What are the upward forces at the girder end points when it is bearing a 3 700 kg shipping container 8.0 m from support A? **(7 marks)**



Pivot A (4 marks for first pivot calculation):

$$\Sigma\tau_{cw} = \Sigma\tau_{ccw} \quad \leftarrow \textbf{1 mark}$$

$$F_C L_C + F_W L_W = F_B L_B$$
$$3700(9.8)(8) + 1200(9.8)(16) = F_B(32) \quad \leftarrow \textbf{2 marks}$$
$$2.90\times10^5 + 1.88\times10^5 = F_B(32)$$
$$1.49\times10^4\text{ N} = F_B \quad \leftarrow \textbf{1 mark}$$

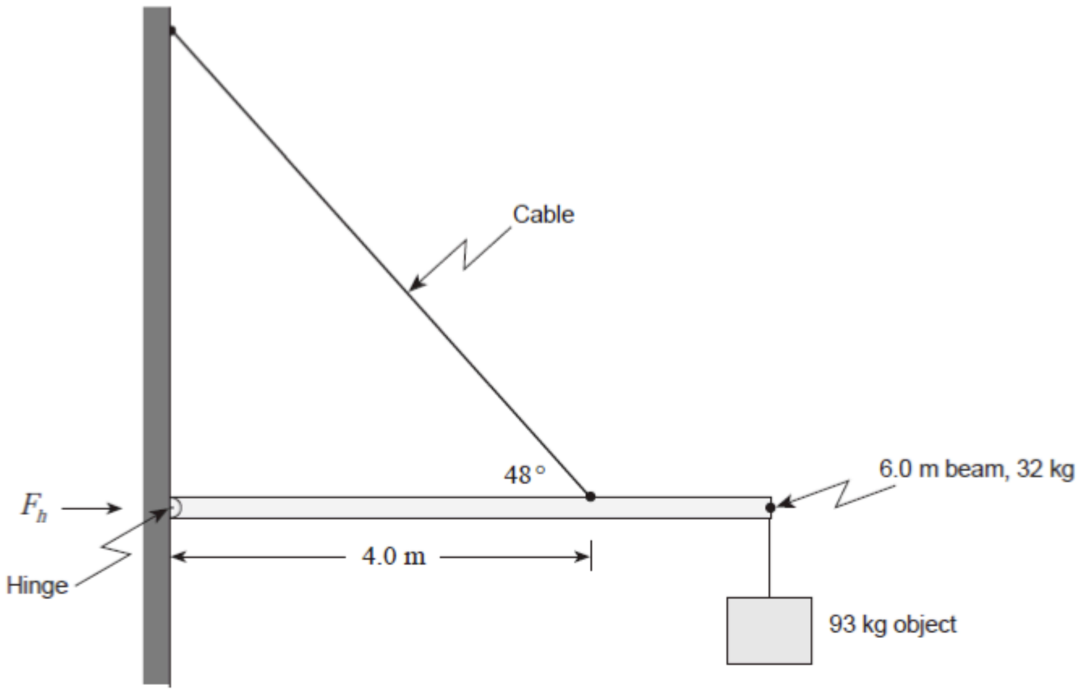
Pivot B (3 marks for second pivot OR sum of forces):

$$F_C L_C + F_W L_W = F_A L_A$$
$$3700(9.8)(24) + 1200(9.8)(16) = F_A(32)$$
$$(8.70\times10^5) + (1.88\times10^5) = F_A(32) \quad \leftarrow \textbf{2 marks}$$
$$3.31\times10^4\text{N} = F_A \quad \leftarrow \textbf{1 mark}$$

Forces:

$$F_C + F_W = F_A + F_B$$
$$3700(9.8) + 1200(9.8) = F_A + F_B \quad \leftarrow \textbf{2 marks}$$
$$(3.63\times10^4) + (1.18\times10^4) = F_A + F_B$$
$$F_A \text{ or } F_B = \quad \leftarrow \textbf{1 mark}$$

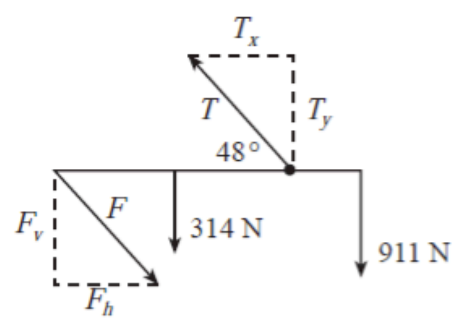
6) A 6.0 m uniform beam of mass 32 kg is suspended horizontally by a hinged end and a cable. A 93 kg object is connected to one end of the beam.



What is the magnitude of the horizontal force F_h that the hinge exerts on the beam? **(7 marks)**

$$\tau_c = \tau_{cc} \text{ about the hinge}$$
$$3.0(314)+6.0(911) = 4.0(T)\sin 48^\circ$$
$$942+5\,470 = 2.97\,T$$
$$2\,160\text{ N} = T$$
← **5 marks**

$$T_x = F_h$$
$$T\cos 48 = F_h$$
$$2\,160\cos 48 = F_h$$
$$F_h = 1\,400\text{ N } (1.4\times10^3\text{ N})$$
← **2 marks**

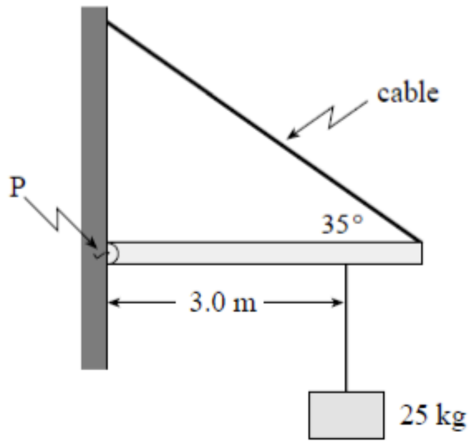


Alternate Solution:

$$\Sigma\tau = 0 \text{ about the hinge}$$
$$(314)(3.0)-T_y(4.0)+911(6.0) = 0$$
$$T_y = 1\,600\text{ N}$$
← **5 marks**

$$F_h = T_x$$
$$F_h = \frac{T_y}{\tan 48}$$
$$F_h = 1\,400\text{ N}\left(1.4\times10^3\text{ N}\right)$$
← **2 marks**

6) A uniform 15 kg beam of length 4.0 m is supported against a wall as shown in the diagram. A 25 kg object is suspended 3.0 m from the hinge P.



a) What is the tension in the support cable? **(5 marks)**

$$\Sigma\tau_p = 0 \qquad \leftarrow \textbf{1 mark}$$

$$W_b \cdot 2.0 + W_1 \cdot 3.0 - T\sin 35^\circ \cdot 4.0 = 0$$

$$T = \frac{W_b \cdot 2.0 + W_1 \cdot 3.0}{\sin 35^\circ \cdot 4.0}$$

$$= \frac{(15\text{ kg}\cdot 9.8\text{ m/s}^2 \cdot 2.0\text{ m}) + (25\text{ kg}\cdot 9.8\text{ m/s}^2\cdot 3.0\text{ m})}{\sin 35^\circ \cdot 4.0\text{ m}}$$

$$= \frac{294\text{ N}\cdot\text{m} + 735\text{ N}\cdot\text{m}}{2.29\text{ m}}$$

← **3 marks**

$$= 4.5\times 10^2\text{ N} \qquad \leftarrow \textbf{1 mark}$$

b) What is the magnitude of the horizontal component of the reaction force of the wall on the beam at the hinge P? **(2 marks)**

$$\Sigma F_x = 0$$

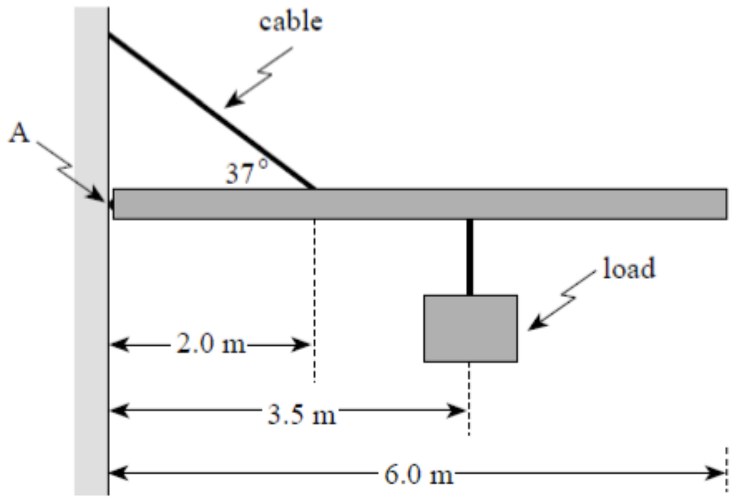
$$\therefore F_{Rx} = T\cos 35^\circ$$

← **1 mark**

$$= 4.5\times 10^2\text{ N}\cdot\cos 35^\circ$$

$$= 3.7\times 10^2\text{ N} \qquad \leftarrow \textbf{1 mark}$$

7) A uniform beam 6.0 m long, and with a mass of 75 kg, is hinged at A. The supporting cable keeps the beam horizontal.



If the maximum tension the cable can withstand is 2.4×10^3 N, what is the maximum mass of the load? **(7 marks)**

$\tau_{CW} = \tau_{CCW}$ ← **1 mark**

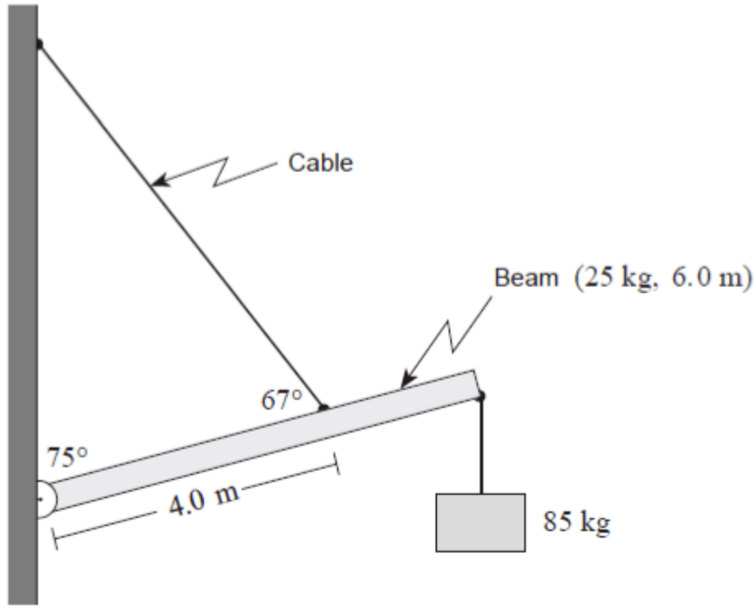
$$\left.\begin{aligned} F\perp &= \left(2.4\times10^3\right)\sin 37° \\ &= 1\ 444.3\ \text{N} \end{aligned}\right\} \leftarrow 2 \textbf{ marks}$$

∴ Using torque about A:

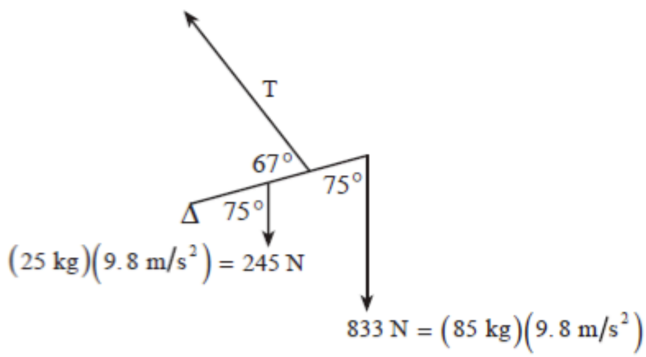
$3.0(735) + 3.5(F_L) = 1\ 444.3(2.0)$ ← 3 **marks**

$$\left.\begin{aligned} 2\ 205 + 3.5(F_L) &= 2\ 888.6\ \text{N} \\ 3.5(F_L) &= 683\ \text{N} \\ \text{Load} &= 195.4\ \text{N} \\ \text{Mass} &= \frac{F_L}{9.8} \\ &= 19.9\ \text{kg} \\ &= 20\ \text{kg} \end{aligned}\right\} \textbf{1 mark}$$

8) A 6.0 m uniform beam of mass 25 kg is suspended by a cable as shown. An 85 kg object hangs from one end.



What is the tension in the cable? **(7 marks)**



$$\Sigma\tau = 0$$

$$\tau_c = \tau_{cc}$$

$$\tau_{245} + \tau_{833} = \tau_T$$ ← **1 mark**

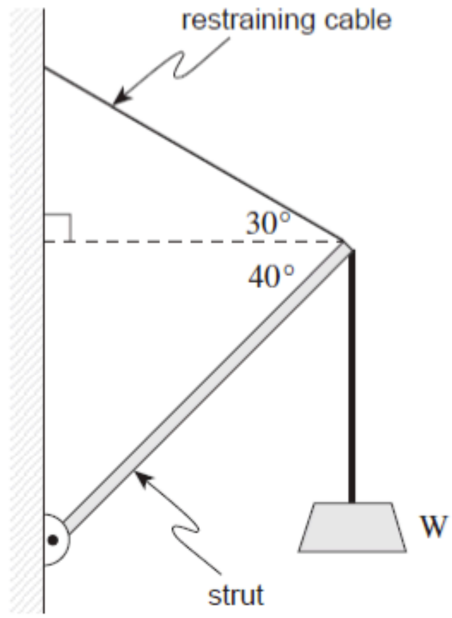
$$3.0\ \text{m}(245\ \text{N})\sin 75° + 6.0\ \text{m}(833\ \text{N})\sin 75° = 4.0\ \text{m}\ T\sin 67°$$ ← **5 marks**

$$710\ \text{N}\cdot\text{m} + 4\,830\ \text{N}\cdot\text{m} = 3.68\ \text{m}\ T$$

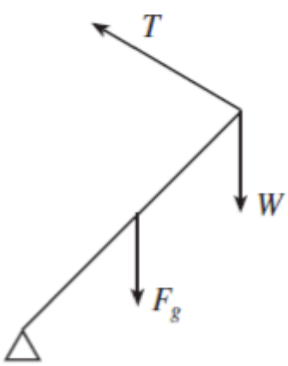
$$5\,540\ \text{N}\cdot\text{m} = 3.68\ \text{m}\ T$$

$$1\,500\ \text{N} = T$$ ← **1 mark**

9) The crane assembly shown in the diagram below consists of a uniform 4.0 m long 65 kg strut and a restraining cable.



What is the maximum weight W that can be supported by this crane if the maximum tension that the restraining cable can withstand is 2400 N? The vertical rope is strong enough to support any required load. **(7 marks)**



$$\Sigma\tau_{pivot} = 0 \quad \text{or} \quad \Sigma\tau_{cw} = \Sigma\tau_{ccw}$$ **$\frac{1}{2}$ mark**

$$W \sin 50 \cdot \cancel{l} + F_g \cdot \sin 50 \cdot \frac{\cancel{l}}{2} = T \sin 70 \cdot \cancel{l}$$ ← **$4\frac{1}{2}$ marks**

$$\therefore W = \frac{T \sin 70 - \frac{F_g \sin 50}{2}}{\sin 50}$$

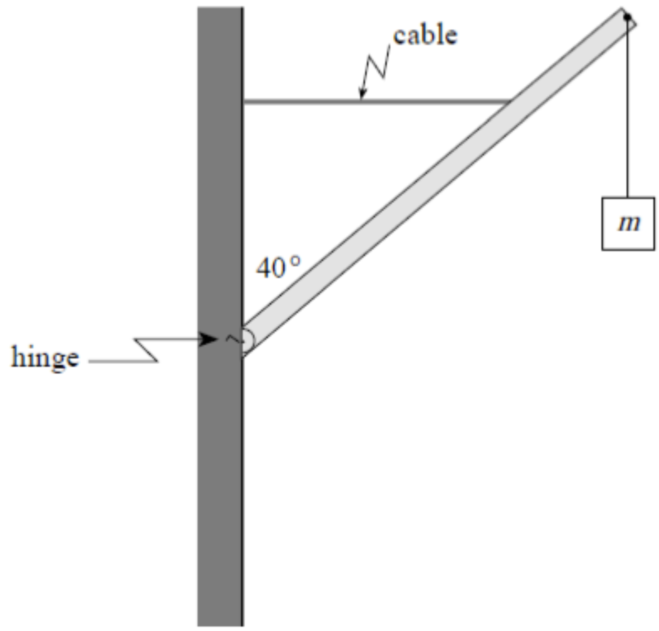
$$= \frac{2\,400 \sin 70 - \frac{65 \cdot 9.8 \cdot \sin 50}{2}}{\sin 50}$$ ← **1 mark**

$$= \frac{2\,255 - 244}{\sin 50}$$

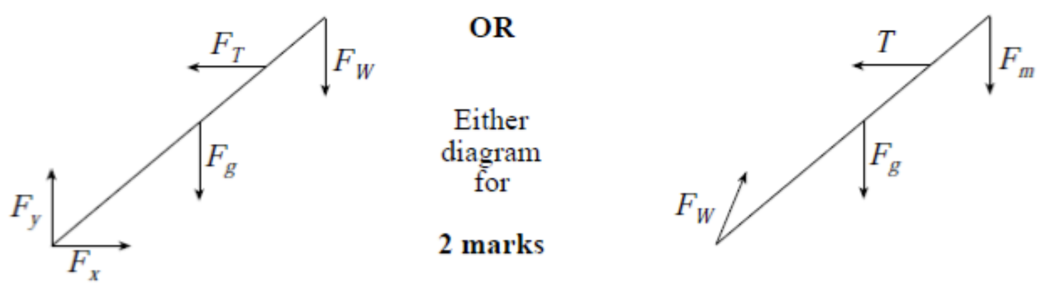
$$= 2\,626 \text{ N}$$

$$= 2.6 \times 10^3 \text{ N}$$ ← **1 mark**

10) A uniform 350 kg beam of length 4.2 m is held stationary by a horizontal cable. The cable is attached to a point on the beam 3.0 m from the hinge.



a) Draw and label a free body diagram showing the forces on the beam. **(2 marks)**



b) If the maximum tension the cable can withstand is 1.3×10^4 N, what maximum mass, ***m***, can be suspended from the end of the beam? **(5 marks)**

$$\tau_{cw} = \tau_{ccw} \quad \leftarrow \textbf{1 mark}$$

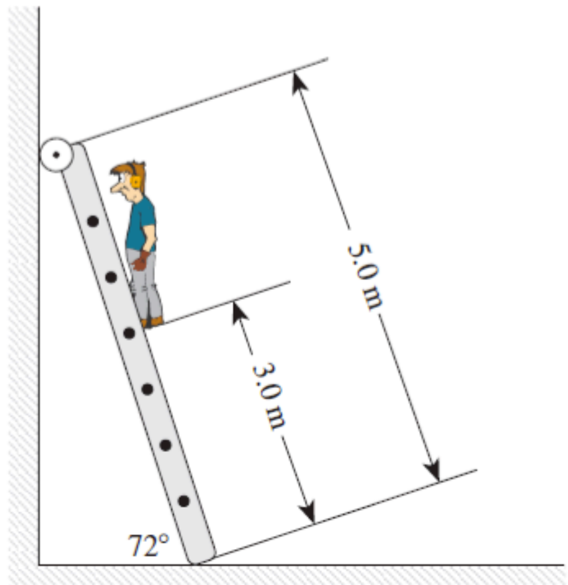
$$F_g(\cos 50)(2.1) + F_W(\cos 50)(4.2) = (1.3 \times 10^4)(\sin 50)(3.0)$$

$$4\ 630 + F_W(\cos 50)(4.2) = 2.99 \times 10^4 \quad \leftarrow \textbf{2 marks}$$

$$F_W = 9.35 \times 10^3 \text{ N} \quad \leftarrow \textbf{1 mark}$$

$$m = \frac{F_W}{g} = 950 \text{ kg} \quad \leftarrow \textbf{1 mark}$$

11) A 65 kg man is 3.0 m up a 5.0 m, 16 kg ladder leaning against a smooth wall at an angle of 72° as shown below.



What minimum force of friction between the ladder and the floor is required to keep the ladder from sliding? **(5 marks)**

$$\Sigma\tau = 0$$

$$\tau_c = \tau_{cc} \qquad \leftarrow \textbf{1 mark}$$

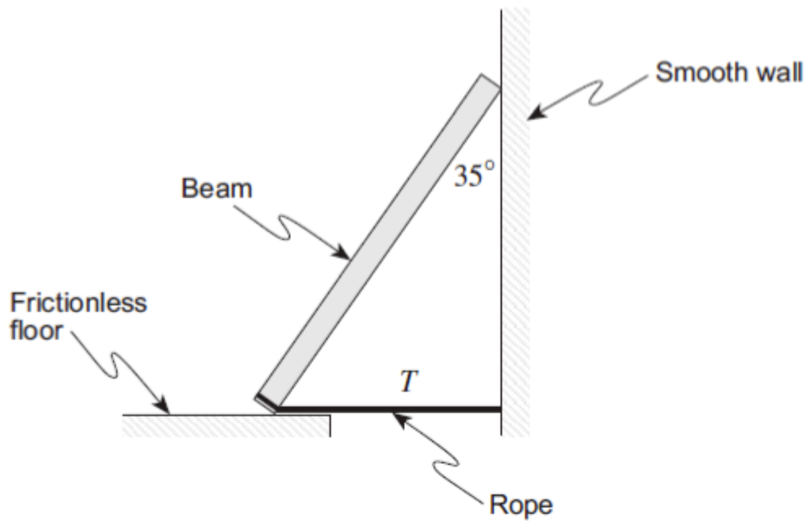
$$F_w \times \sin 72° \times 5 = 65 \times 9.8 \times \sin 18° \times 3 + 16 \times 9.8 \times \sin 18° \times 2.5 \qquad \leftarrow \textbf{2 marks}$$

$$F_w \times 4.755 = 590.53 + 121.13 \qquad \leftarrow \textbf{1 mark}$$

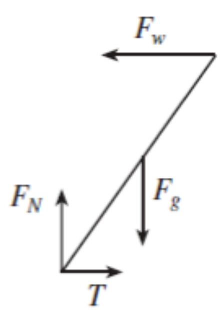
$$F_f = F_w = \frac{711.66}{4.755}$$

$$\therefore F_f = 150 \text{ N} \qquad \leftarrow \textbf{1 mark}$$

12) A 24 kg beam of length 2.4 m leans against a smooth wall. A horizontal rope tied to the wall and the beam holds the beam on a frictionless floor as shown.

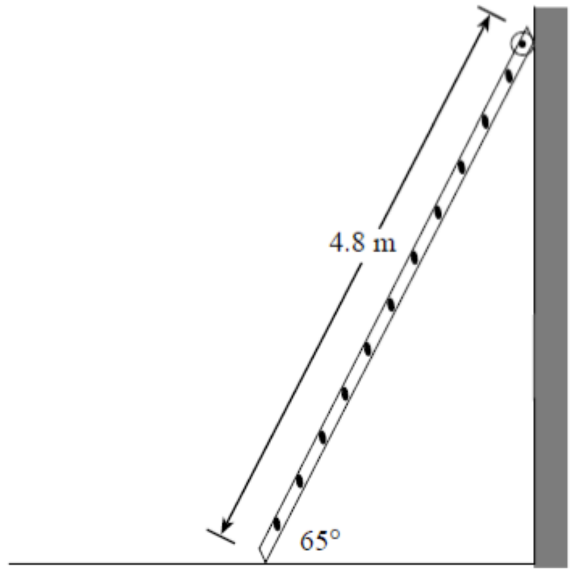


a) Draw a labelled free-body diagram for the forces acting on the beam. **(2 marks)**

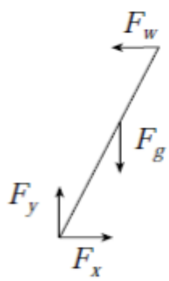


b) What is the tension in the rope? **(5 marks)**

13) A uniform 4.8 m long ladder of mass 16 kg leans against a **frictionless** vertical wall as shown in the diagram below.



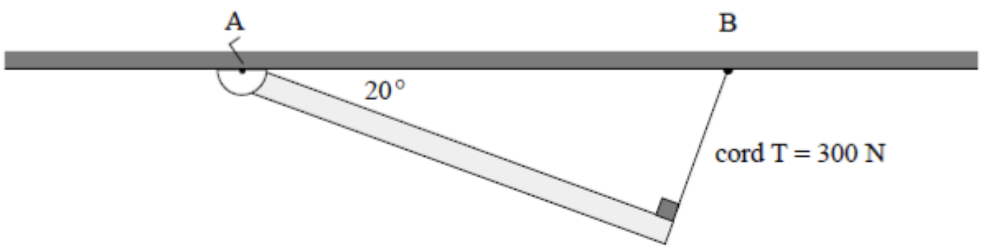
a) Draw and label a free body diagram showing the forces acting on the ladder. **(2 marks)**



2 marks

b) What minimum force of friction is needed at the base of the ladder to keep it from sliding? **(5 marks)**

14) A 3.8 m uniform beam is attached to the ceiling with a hinge at A and a cord with a tension of 300 N at B.



Determine the mass of the beam. **(7 marks)**

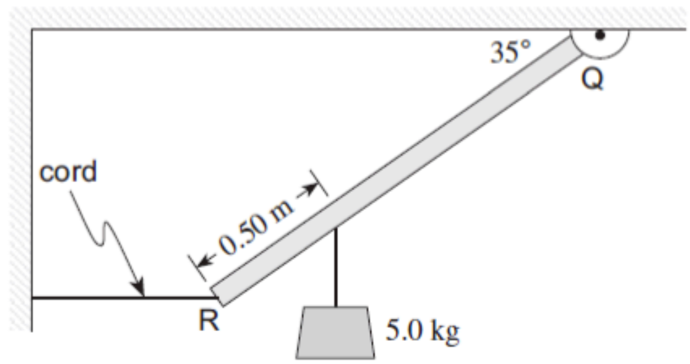
$$\tau_c = \tau_{cc}$$ ← **1 mark**

$$\frac{\ell}{2} F \sin\theta = \ell F \sin\theta$$ ← **3 marks**

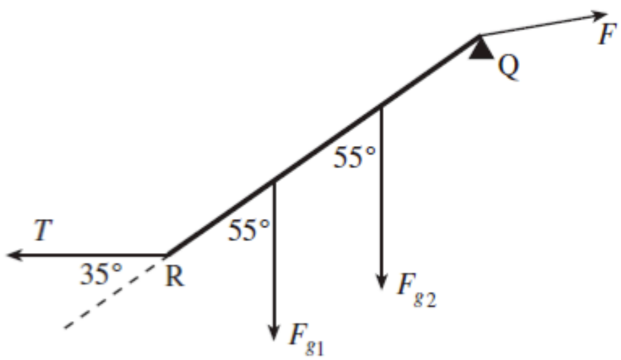
$$1.9(9.8\ m)\sin 70° = 3.8(300)$$ ← **2 marks**

$$m = 65 \text{ kg}$$ ← **1 mark**

15) A uniform 2.4 m beam RQ has a mass of 3.0 kg. The beam is hinged at Q and held in place by a horizontal cord attached at R. A 5.0 kg mass is suspended 0.50 m from R.



What is the tension in the horizontal cord? **(7 marks)**



$$\Sigma\tau_Q = 0$$

$$\Sigma\tau_{cw_Q} = \Sigma\tau_{ccw_Q}$$ ← **$1\frac{1}{2}$ marks**

$$T\sin 35\cdot 2.4 = F_{g_1}\cdot\sin 55\cdot 1.9 + F_{g_2}\cdot\sin 55\cdot 1.2$$

$$T\sin 35\cdot 2.4 = 5.0\cdot 9.8\cdot\sin 55\cdot 1.9 + 3.0\cdot 9.8\cdot\sin 55\cdot 1.2$$

$$\therefore T = \frac{5.0\cdot 9.8\cdot\sin 55\cdot 1.9 + 3.0\cdot 9.8\cdot\sin 55\cdot 1.2}{\sin 35\cdot 2.4}$$

← **$4\frac{1}{2}$ marks**

$$= 76 \text{ N}$$ ← **1 mark**