MAT 2379 3X (Spring 2015)
Assignment 1 - Solutions

[4] 1. Let A and B be the events that the white rate is infected by a virus A and by a virus B, respectively. We know that $P(A) = 0.75$, $P(B) = 0.7$ and $P(A \cap B) = 0.5$.

(a) The probability that a randomly chosen rat infected by at least one of the two viruses is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.75 + 0.7 - 0.5 = 0.95.$$

(b) The probability that a randomly chosen rat becomes infected by virus A but not virus B is

$$P(A \cap B') = P(A) - P(A \cap B) = 0.75 - 0.5 = 0.25.$$

(c) The probability that a randomly chosen rat becomes infected by a virus but not both is

$$P(A \cap B') + P(A' \cap B) = 0.25 + (0.7 - 0.5) = 0.45.$$

Marking scheme: 4 points : part a and b each 1, part c 2 points

[3] 2. Let A be the event that an individual from this population has the disease A and let B be the event that an individual from this population has the disease B. We know that $P(A) = 0.15$, $P(B) = 0.2$ and $P(A \cup B) = 0.25$.

(a) The probability that this person contracted both diseases is

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.15 + 0.2 - 0.25 = 0.1.$$

(b) Given that the chosen individual has at least one of the diseases, the probability that this person has contracted both diseases is

$$P(A \cap B | A \cup B) = \frac{P[(A \cup B) \cap (A \cap B))]}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.1}{0.25} = 0.4.$$

Note that if a person has both diseases and the person has at least one of the two diseases, then it should be obvious that this is equivalent to this person having both diseases. *Hint:* Draw a Venn diagram.

Marking scheme: 3 points : part a , part b 2 points

[3] 3. Let M be the event of being a man and W be the event of being a woman. Let C be the event of being colour-blind. We know that $P(M) = P(F) = 0.5$ and $P(C|M) = 0.07$ and $P(C|W) = 0.0025$. We want

$$\begin{aligned} P(M|C) &= \frac{P(M \cap C)}{P(C)} \\ &= \frac{P(C|M)P(M)}{P(C|M)P(M) + P(C|W)P(W)} \\ &= \frac{(0.07)(0.5)}{(0.07)(0.5) + (0.0025)(0.5)} = 0.9655. \end{aligned}$$

[6] 4. Let F be the event of having an alcoholic father and M the event of having an alcoholic mother. We know that $P(F) = 0.42$, $P(M) = 0.08$ and $P(F \cup M) = 0.48$.

(a) The probability of having two alcoholic parents is

$$P(F \cap M) = P(F) + P(M) - P(F \cap M) = 0.42 + 0.08 - 0.48 = 0.02.$$

(b) The probability of having an alcoholic mother but not an alcoholic father is

$$P(M \cap F') = P(M) - P(M \cap F) = 0.08 - 0.02 = 0.06.$$

(c) The probability of having an alcoholic mother, if they have an alcoholic father is

$$P(M|F) = \frac{P(M \cap F)}{P(F)} = \frac{0.02}{0.42} = 0.0476.$$

(d) The probability of having an alcoholic mother, if they do not have an alcoholic father is

$$P(M|F') = \frac{P(M \cap F')}{P(F')} = \frac{0.06}{1 - 0.42} = 0.1034.$$

Marking scheme: 6 points : part a and b each 1 point, part c, d each 2 points

[4] 5. Let the events A, B, C be that the machine was manufactured by company A, by company B and by company C, respectively. Let D be the event that the machine is defective. We have $P(A) = 0.8$, $P(B) = 0.15$ and $P(C) = 0.05$. Furthermore, the defective rates are $P(D|A) = 0.04$, $P(D|B) = 0.05$ and $P(D|C) = 0.08$.

(a) Given that the machine is defective, the probability that it was made by A is

$$\begin{aligned} P(A|D) &= \frac{P(A \cap D)}{P(D)} \\ &= \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)} \\ &= \frac{(0.04)(0.8)}{(0.04)(0.8) + (0.05)(0.15) + (0.08)(0.05)} \\ &= 0.7356. \end{aligned}$$

is defective is

$$P(A \cap D) = P(D|A)P(A) = (0.04)(0.8) = 0.032.$$

(b) The probability that it was made by company A and it is not defective is

$$P(A \cap D') = P(D'|A)P(A) = (1 - 0.04)(0.8) = 0.768,$$

or alternatively we could have computed the probability as follows:

$$P(A \cap D') = P(A) - P(A \cap D) = 0.8 - 0.032 = 0.768.$$

Marking scheme: 4 points : part a and b each 2 points

[/**20**]