

COMM 215: BUSINESS STATISTICS

SOLUTIONS TO REVIEW PROBLEMS 1

Descriptive Statistics

1. a.

Leaf-unit = 0.01

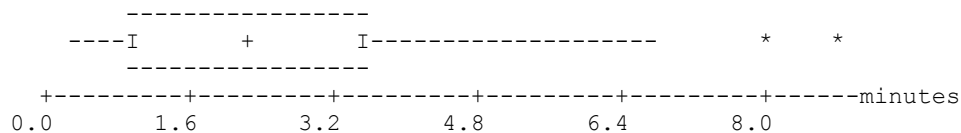
0	26, 45, 52, 81, 85, 88, 91, 94
1	01, 22, 42, 91, 92, 94
2	25, 33, 51, 62, 66, 93, 95, 95
3	51, 60, 72
4	91
5	19
6	71
7	96
8	81

Leaf Unit = 0.10

0	24588899
1	024999
2	23566999
3	567
4	9
5	1
6	7
7	9
8	8

- b. i. $\bar{X} = 2.689$ Median = 2.29 Mode = 2.95
- ii. $Q1 = 31 (0.25) = 7.75$; $(.94-.91) (.75) = 0.0225$; $Q1 = 0.9325$
 $Q3 = 31 (.75) = 23.25$; $(3.60-3.51) (.25) = 0.0225$; $Q3 = 3.532$
- iii. Range = $8.81 - 0.26 = 8.55$, IQR = $3.5325 - 0.9325 = 2.6$
- iv. $S^2 = 4.69$, $S = 2.166$
- c. $1.5 (IQR)$, $1.5 (2.6) = 3.9$, $Q1 - 3.9 = 0$, $Q3 + 3.9 = 7.4$

Two outliers whose values are 7.96 and 8.81



2. a.

Leaf Unit = 1.0

0	8
1	0 5
2	2 4 5
3	0 5 5 8
4	03

b. $X_{\min} = 8, \quad Q1 = 16.75, \quad Q2 = 27.5, \quad Q3 = 37.25, \quad X_{\max} = 43$

c. $\bar{X} = 27.083, \quad S = 11.735$

d. $L_{40} = 13 (.40) = 5.2; \quad 40^{\text{th}} \text{ Percentile} = 24.2$

3. a.

0	3		0	3 5 6 8 9
0	5 6 8 9		1	0 0 2 4 4 8
1	1 0 0 2 4 4	or	2	0 1 4 5
1	8		3	3
2	0 1 4			
2	5			
3	3			

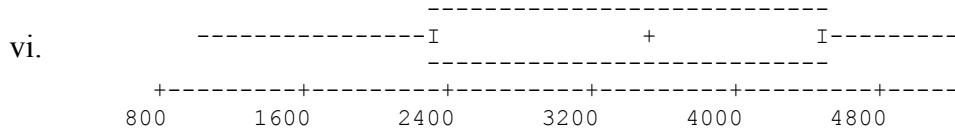
b. 30th percentile: $17 (.3) = 9.1$
70th percentile: $17 (.7) = 18.8$
IQR = 12.5
 $1.5(\text{IQR}) = 1.5 (12.5) = 18.75$
 $Q3 + 1.5(\text{IQR}) = 20.75 + 18.75 = 39.5$

c. $Q1 = 8.25, \quad Q3 = 20.75$
Yes, 42 days would be considered an outlier.

4. a.

- i. $\bar{X} = \$3430, \quad \text{Median} = \$3500, \quad \text{Mode} = \3500
- ii. $Q1 = \$2,275, \quad Q3 = \$4,525$
- iii. $\text{Range} = 5200 - 1000 = \$4,200; \quad \text{IQR} = 4525 - 2275 = \$2,250$
- iv. $S^2 = 1,560,105; \quad S = \$1,249$

v.



Compute: $(1.5) IQR + Q3 = 3375 + 4525 = \7900 . No outliers

- b. C.V. retainers = $(1249/3430) * 100 = 36.4\%$
 C.V total compensation = $(20,000/120,000) * 100 = 16.67\%$
 No, each retainer is relatively more variable.

5. a.

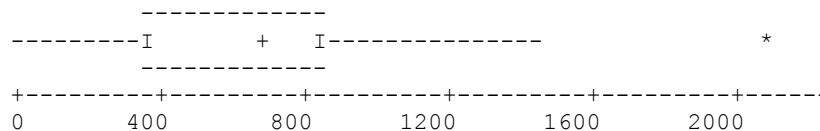
	Fi	Cumulative	Cumulative relative
20-24	3	3	0.05
25-29	7	10	0.167
30-34	11	21	0.35
35-39	13	34	0.57
40-44	9	43	0.717
45-49	7	50	0.833
50-54	5	55	0.917
55-59	3	58	0.967
60-64	1	59	0.983
65-69	1	60	1

b. Right or positively skewed.

6. a. $\bar{X} = \$2492.125$ $S = \$273.1475$ $CV = 10.96\%$
 b. The median = \$2405; 40th percentile: $9(.4) = 3.6 = \$2390$
 c. $CV = 320/3600 = 0.0889 * 100 = 8.89\%$
 No, they are less variable or smaller C.V.

7. a. $\bar{X} = 696.809$ $S = 483.188$

b.



$IQR = 907.5 - 339.5 = 568$ $IQR (1.5) = 852$, $907.5 + 852 = 1759.5$
 Therefore, the data value 2063 is an outlier.

- c. Point where 50% of the observations are more than \$684 and 50% are less
- d. The proportion below $\bar{X} - 1S = 213.6$ plus the proportion above $\bar{X} + 2S = 1663.2$
 You would expect: $0.34 + 0.475 = 0.815$ or 81.5%
 Actually proportion: out of 21, there are a total of 17 in the two regions giving $17/21 = 0.809$ or 80.9%.

8.

a.

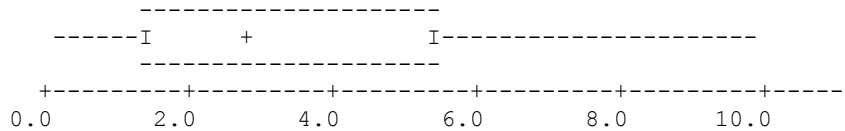
Leaf Unit = 0.10

0	2455789
1	124589
2	34457889
3	13347
4	247
5	2568
6	238
7	68
8	
9	57

b.

$X_{\text{minimum}} = 0.2$;
 $Q1 = 1.425$;
 $Q2 = 2.85$;
 $Q3 = 5.425$
 $X_{\text{maximum}} = 9.7$

c.



$IQR (1.5) = (5.425 - 1.425) 1.5 = 6$; $6 + 5.425 = 11.425$
 There are no outliers.

d. Distribution is positive or right skewed.

9. a. $Cov(x,y) = -15.133$ indicating a negative or an inverse linear relationship between price and demand
- b. $r = -15.133 / ((1.94) (8.214)) = -0.9496$, indicating a strong inverse or negative linear association between price and demand.

10. Nonsampling errors occur due to incorrect acquisitions of observations, improper selections samples, and the lack of response from individual in the sample. Examples are:
1. data acquisitions: wrong measurements on employees information (*answers may vary*)
 2. exclusion of part-time employees from the sample (*answers may vary*)
 3. missing information on employees characteristics; responses coming only from people who have strong feeling about the issues (*answers may vary*)

Probability

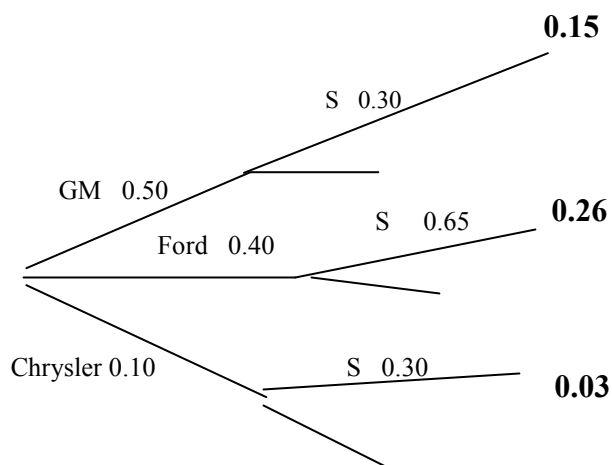
1.

	E	U	
M	.52	.08	.60
W	.28	.12	.40
	.80	.20	

$$\begin{aligned}
 .6 U &= WU \\
 .6 (.2) &= .12 \\
 .3 W &= WU, \\
 .3 W &= .12, \\
 W &= .4
 \end{aligned}$$

- a. $P(W) = 0.40$
- b. $P(U/M) = 0.08 / 0.60 = .13$

2.

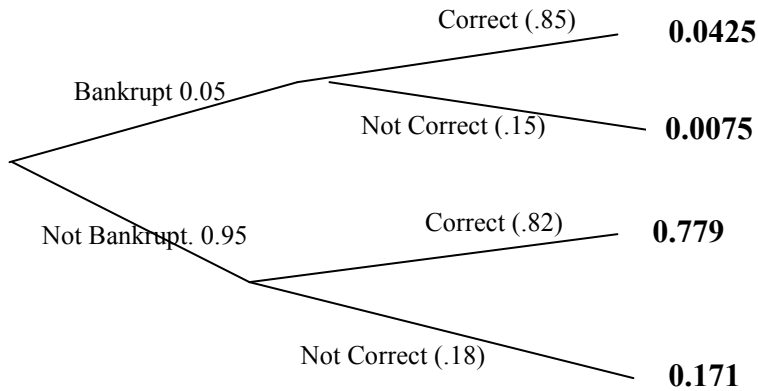


$$P(\text{Strike}) = .15 + .26 + .03 = .44$$

3.

- a. $(.9)(.9) = .81$
- b. $(.1)(.1) = .01$
- c. $(.9)(.1) + (.1)(.9) + (.9)(.9) = .99$ or $1 - 0.01 = 0.99$

4.

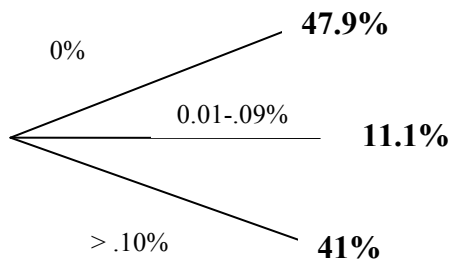


$$\begin{aligned}
 P(\text{correct}) &= 0.0425 + 0.779 \\
 &= 0.8215 \\
 0.8215 * 100 \text{ firms} &= 82 \\
 &\text{firms}
 \end{aligned}$$

5.

- a. $P(<30 \text{ or } 1^{\text{st}}) = 1 - (.07 + .09 + .05) = .79$
- b. $P(\text{repeat} \mid < 30) = (.01 + .02 + .04) / .43 = .16$
- c. $P(\geq 30 \mid 1^{\text{st}}) = .36 / .72 = .50$
 $P(\geq 30 \mid \text{repeat}) = .21 / .28 = .75$; repeat offender is more likely to be ≥ 30
- d. 50th p of first offenders = .36 = 30
 a. 50th p of repeat offenders = .14 = 35, repeat offenders are older
- e. $(.28)(.28) = .0784$

6.



- a. $P(X \geq .01) = 11.1 + 41 = 52.1\%$
- b. $P(X > 0.1 \mid X \geq 0.01) = .41 / .521 = .7869$

7. a. i. $365/500 = .73$ iii. $30/500 = .06$
 ii. $110/340 = .3235$ iv. $\frac{230+140-30}{500} = .68$

b. Not mutually exclusive since $P(\text{work pt-time and } < 6) = 60/500 = .12$.
 It will be zero if mutually exclusive.

Independent if $P(\text{work pt. time}) * P(<6) = P(\text{work pt. time and } < 6)$
 Since $135/500 * 140/500 \neq .12$, the two are not independent.

8.

	P	S	A	
U	0.125	0	0.06	0.185
U ^c	0.125	0.15	0.54	0.815
	0.25	0.15	0.60	1

- a. $P(U) = 0.185$
 b. $P(A | U^c) = .54/.815 = .6625$

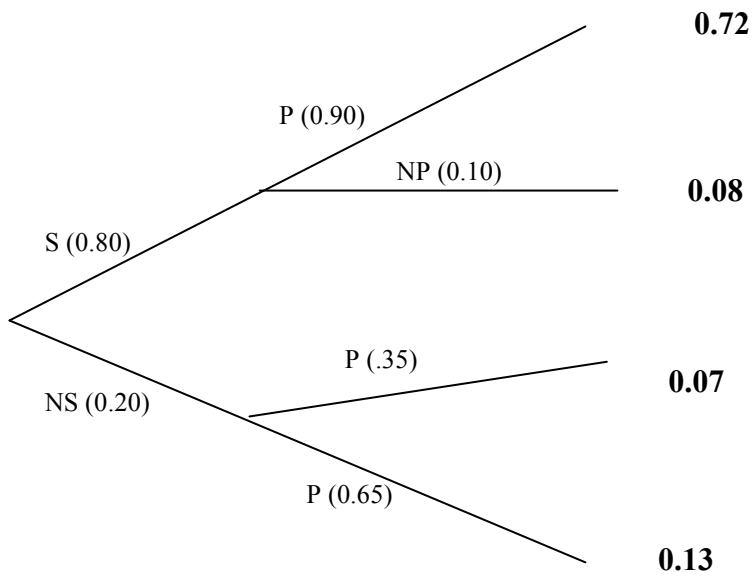
9.

a.

Own Stock	City			Total
	A	B	C	
YES	85 (.17)	55 (.11)	50 (.10)	190 (.38)
NO	115 (.23)	95 (.19)	100 (.20)	310 (.62)
Total	200 (.40)	150 (.30)	150 (.30)	500 (1.0)

- b. i. $P(\text{YES}) = 0.38$; ii. $P(\text{YES and A}) = 85/500 = 0.17$;
 iii. $P(\text{B and NO}) = 95/500 = 0.19$; iv. $P(\text{B and YES}) = 215/500 = 0.43$
 c. $P(\text{NO / NOT B}) = 215/350 = 0.614$

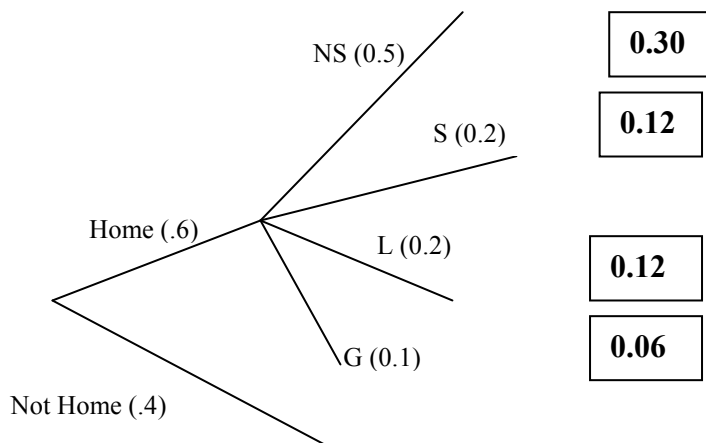
10.



a. $P(S/P) = .72/.79 = 0.91$

b. $P(NS/NP) = 0.61$

11.



a. $0.4 + (.6)(.5) = .70$

b. $0.12 + .06 = .18$

12.

a.

	Defective (D)	Not Defective (ND)	
1	.008	.392	.40
2	.0075	.2925	.30
3	.0045	.2955	.30
	.02	.98	

- b. $P(1/D) = .008/.02 = .40$
c. $P(D \text{ or } 2) = .02 + .03 - .0075 = .3125$
d. $P(D) = .02$
e. $P(2/D) = P(2) = .0075/.02 = .375 = .30$; not independent

13. a.

	E	U	
M	.5	.04	.54
F	.43	.03	.46
	.93	.07	1

- b. $P(F/U) = .03/.07 = .428$
c. i. $P(\text{um}*\text{um}) = (.04)(.04) = .0016$ ii. $P(1 \text{ or } 2)$

Easiest way would be with Binomial (but not the only way)

$$(\text{Combination of 2 and 1}) (.04) (.96) + (\text{Combination of 2 and 2}) (.04)^2 (.96)^0 = .0784$$

- d. $P(U) = .07$
e. $P(\text{Males}) = .54$ $P(\text{Male}/U) = 0.04/0.07$ $.54 \neq .571$

Therefore they are not independent

(Can use any of the formulas as long as it is correct)

$$P(A \cap B) = P(A) * P(B) \quad P(A/B) = \frac{P(A \cap B)}{P(B)} \quad P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Discrete Probability Distribution / Binomial Distribution

1. a.

	1st	2nd	3rd	Total
Regular	.9 (.2)=0.18 \$4.00	.7 (.3)=0.21 \$2.00	.8(.5)=0.4 \$0.50	0.79
Reduced	.1 (.2)=0.02 \$2.00	.3 (.3)=0.09 \$1.00	.2 (.5)=0.10 \$0.25	0.21
Total	0.20	0.30	0.50	1

$$P(\text{regular}) = .79$$

b. $P(1^{\text{st}}/\text{regular}) = .18/.79 = .227$

c. $\mu = .18(4) + .21(2) + .4(.50) + .02(2) + .09(1) + .10(.25)$
 $= \$1.495$

$$\mu_{\text{regular}} = \frac{.18(4) + .21(2) + .4(.5)}{.79} = \$1.696$$

d. $P(X \geq 2 / n = 8, p = .21) = 1 - \{f(0) + f(1)\}$

$$P(0) = (\text{Combination of 8 and 0}) (.21)^0 (.79)^8 = .1517$$

$$P(1) = (\text{Combination of 8 and 1}) (.21)^1 (.79)^7 = .3226$$

$$1 - (.1517 + .3226) = .5257$$

$$E(X) = 8(.21) = 1.68; \quad \sigma = \sqrt{1.68(.79)} = 1.15$$

e. $\$1.495(75000) = \$112,125$ or $\$1.50(75000) = \$112,500$

2. a. $X = \text{arrive on time } p = .75$
 $P(X = 4 / n = 4, p = .75) = .3164$

b. $P(X = 3) + P(X = 4) = .4219 + .3164 = 0.7383$

3. $.05(2) + .15(5) + .25(10) + .35(20) + .10(50) + .10(100)$

$$\mu = \sum Xi * P(X) = \$25.35, \quad \sigma^2 = \sum (X - \mu)^2 P(X) = 776.33$$

4. a. $\mu = np = 12(.30) = 3.6 \text{ people}$ $\mu = np = 12(.30) = 3.6 \text{ people}$

b. $P(X \geq 3 / n = 12, p = .30) = 1 - (f(0) + f(1) + f(2))$
 $1 - (.0138 + .0712 + .1678) = .7472$

c. $P(X \geq 4 / n = 12, p = .30) = 1 - (f(0) + f(1) + f(2) + f(3))$
 $1 - .4925 = 0.5075$

5. a. $E(X) = .3(100) + .2(100) + .7(100) = \120
 b. $\text{Var}(X) = \sum (X - \mu)^2 P(x) = \480.00

6. a. $\sum (X - \mu)^2 P(x) = \480.00
 $P(X=3 / n = 15, p = .05) = (\text{Combination of 15 and 3}) (.05)^3 (.95)^{12} = .0307$
 b. $P(X \geq 3 / n = 15, p = .05) = 1 - (f(0) + f(1) + f(2)) = 0.0361$
 c. If P is actually .05, then $P(X \geq 3)$ is very small therefore it is unlikely.

7. a. $E(X) = \mu = 10(.2) = 2$; standard deviation = $\sqrt{10(.2)(.8)} = 1.6$
 b. $P(X \leq 2 / n = 10, p = .2) = .1074 + .2684 + .3020 = .6778$
 c. $P(X=3 / n = 10, p = .8) = (\text{Combination of 10 and 3}) (.8)^3 (.20)^7 = .000786$
 d. $p = .2/(1-.2) = 0.25$ $P(X \leq 2 / n = 9, p = .25) = 0.60$

8. a. Mean = $\sum Xp(x) = 1.98$
 b. $\text{Var}(x) = \sum (x - \mu)^2 p(x) = 1.919$ Standard deviation = 1.385
 c. Expected Profit = $1.98(2000) = \$3,960$
 d. $P(X_1 + X_2 < 3) = P\{(0,0), (0,2), (1,0), (1,1), (2,0)\}$
 $= (.15)(.15) + (.15)(.24) + (.15 + .31) + (.24)(.15) + (.24)(.24) + (.31)(.15) = .2451$

9. a. $P(X = 6 / n = 12, p = .30) = .0792$
 b. $P(X \geq 6) = 1 - P(X \leq 5) = .1179$
 c. $P(X \leq 2) = f(0) + f(1) + f(2) = .2528$
 d. $E(X) = np = 12(.3) = 3.6$, or $3.6(100) = \$360$

10. a. $E(X) = 0(.48) + 1(.2) + 2(.15) + 3(.08) + 4(.05) + 5(.03) + 6(.01) = 1.15$, $\text{Var}(X) = 2.1075$
 b. $f(2) + f(3) = .15 + .08 = .23$
 c. $P(X \geq 2) = 1 - (.48 + .20) = .32$
 d. $P(X < 3) = P(0,0) + P(0,1) + P(0,2) + P(1,0) + P(1,1) + P(2,0) = .6064$

11. a. $P(X \geq 3) = 1 - \{f(0) + f(1) + f(2)\} = 0.74$
 b. $P(2 \leq X \leq 5) = f(2) + f(3) + f(4) + f(5) = 0.67$
 c. $E(X) = 0(.05) + 1(.08) + 2(.13) + 3(.23) + 4(.18) + 5(.13) + 6(.08) + 7(.06) + 8(.06) = 3.78$
 d. $\sigma = \sqrt{4.2} = 2.04$

12. a. $E(X) = 0(.1) + 1000(.3) + 2000(.4) + 3000(.2) = \1700

b.

Sales (x)	Net Profit	P(X)	
0	-28500	.1	-2850
1000	-12500	.3	-3750
2000	3500	.4	1400
3000	19500	.2	3900
Expected Net Profit			\$-1300

13.

- $P(X \geq 9 / n = 15, p = .7) = 1 - P(X \leq 8) = 1 - .131 = 0.869$
- $P(X \geq 12 / n = 15, p = .7) = 1 - P(X \leq 11) = 1 - .703 = 0.297$
- $P(X \leq 2 / n = 15, p = .25) = p(0) + p(1) + p(2)$
 $= .0134 + .0668 + .1559 = 0.236$
- $0.297 (250) = 74.25$

14.

- $p(1) + p(4) + p(5) = .25 + .15 + .10 = 0.5$
- $1(.25) + 2(.33) + 3(.17) + 4(.15) + 5(.10) = 2.52$
- $\sigma = \sqrt{1.649} = 1.28$

15.

- $P(X=10 / n = 10, p = .6) = (\text{Combination of 10 and 10}) (.6)^{10} (.4)^0 = .006$
- $P(X \leq 2) = p(0) + p(1) + p(2) = .0001 + .0016 + .0106 = .0123$

Normal Probability Distributions

- 1 a. $p(x \leq 60) = p\left(z \leq \frac{60-80}{10}\right) = p(z \leq -2) = .0228$
- b. $p(60 \leq x \leq 75) = p\left(\frac{60-80}{10} \leq z \leq \frac{75-80}{10}\right) = p(-2 \leq z \leq -0.5)$
- c. $p(x \geq 90) = p\left(z \geq \frac{90-80}{10}\right) = p(z \geq 1) = 0.1587 \times 60 = 9.5 \approx 10 \text{ student}$
- 2 a. $p(x \geq 4) = p\left(z \geq \frac{4-4.5}{0.6}\right) = p(z \geq -0.833) = 0.5 + 0.2967$
 $0.7967 \times 80 = 63.7 \approx 64$
- b. $\frac{10}{80} = 0.125$, $p(z \geq z_0) = .125 \rightarrow z_0 = 1.15$
 $4.5 + 1.15(0.6) = 5.19$
- c. $p\left(z \geq \frac{5.1-4.5}{0.6}\right) = p(z \geq 1)$ and $p\left(z \geq \frac{4.5-4.5}{0.6}\right)$
 $p\left(\frac{z \geq 1}{z \geq 0}\right) = \frac{.1587}{.5} = 0.317$
- 3 a. $p(x > 36.000) \text{ or } p(x < 33.500) = p\left(z > \frac{36.000-34.265}{2000}\right) +$
 $p\left(z < \frac{33.500-34.265}{2000}\right) = p(z > 0.861) + p(z < -0.383)$
 $0.192 + 0.352 = 0.544$
- b. $p\left(z > \frac{36.000-34.265}{2000}\right) \times p\left(z > \frac{36.000-35.185}{2400}\right)$
 $p(z \geq 0.868) \times p(z > 0.34) = (.192)(.3669) = 0.07$
- c. $p(x \leq x_0) = .90$ $1.28 = \frac{x-35.185}{2400}$ $x = \$38.257$
- 4 a. $1 - p(18 \leq x \leq 26) = 1 - \left(\frac{18-20}{5} \leq z \leq \frac{26-20}{5}\right)$
 $1 - p(-0.4 \leq z \leq 1.2) = 1 - (0.3849 + 0.1554) = 0.459$
- b. $p(x \leq 7) = p\left(z \leq \frac{7-20}{5}\right) = -2.6$ $p(z \leq -2.6) = 0.0047$
 $\frac{p(x \leq 7)}{p(x \leq 20)} = \frac{0.0047}{0.5} = 0.0094$

4. c. $p(x \geq 1) = 1 - p(x = 0)$

$$p(x = 0) = \frac{10!}{0!10!} (.54)^0 (.46)^{10} = 0.000424$$

$$1 - 0.000424 = 0.999576$$

d. $p(x \leq 15) = p\left(z \leq \frac{15 - 20}{5}\right) = p(z \leq -1) = 0.5 - 0.3413$

$$= 0.1587 \times 200 = 31.74 \approx 32$$

5. a. $p(x > 140.5) = p\left(z > \frac{140.5 - 140}{0.2}\right) = p(z > 2.5) =$

$$.05 - 0.4938 = 0.0062$$

b. $p(140.2 \leq x \leq 140.5) = p\left(z \left(\frac{140.2 - 140}{0.2}\right) \leq z \leq \frac{140.5 - 140}{0.2}\right) = 1.0$

$$p(1.0 \leq z \leq 2.5) = 0.4938 - 0.3413 = 0.1525$$

c. $p(x < 139.8) \text{ or } p(x > 140.2)$

$$p(z < -1) \text{ or } p(z > +1) = 0.5 - 0.3413 = 0.1587 \times 2 = 0.3174$$

d. $p(x = 0 / n = 15, p = 0.3174) = \frac{15!}{0!15!} (0.3174)^0 (0.6826)^{15} = 0.0033$

e. $p(x = 5 / n = 15, p = 0.3174) = \frac{15!}{5!10!} (0.3174)^5 (0.6826)^{10} = 0.2110$

6. a. $p(x \leq 6) = p\left(z \left(\frac{6 - 12}{3}\right)\right) = -2.0 \quad p(z \leq -2) = 0.5 - 0.4772 = 0.0228$

b. $p(x > 15) = p\left(z > \frac{15 - 12}{3}\right) = p(z > 1.0) = 0.5 - 0.3413 = 0.1587$

c. $z = \frac{x - \mu}{\sigma} \quad 1.28 = \frac{x - 12}{3} \quad x = 15.84 \approx 16 \text{ weeks}$

7. a. $p(x > 214) = p\left(z > \frac{214 - 250}{45}\right) = p(z > -0.8) = 0.2881 + 0.5 = 0.7881$

b. $p(x \leq 200) = p\left(z \leq \frac{200 - 250}{45}\right) = p(z \leq -1.11) = 0.5 - 0.3665 = 0.1335$

Thus, 11.35% of 52 weeks. That is, $0.1335 \times 52 \text{ weeks} = 6.9 \approx 7 \text{ weeks}$

c. Find the 60th percentile, such that $P(Z \leq z^*) = 0.6$,

where $z^* = \frac{x^* - 250}{45}$. From table, $z^* = 0.2533$. Then, $0.2533 = \frac{x^* - 250}{45}$,

and solving for $x^* = 0.2533 \times 45 + 250 = 261.4006$