Midterm 1
Econ 2222A
October 8, 2014, 7-9pm

Instructions:

- Show your work
- Express numeric answers in either decimal form or simplified (that is, "reduced") fraction form. Non-reduced fractions may be marked as incorrect.
- Use of only non-graphic (i.e. non-programmable) calculators is allowed.
- All exam materials, INCLUDING THIS SHEET OF PAPER, must be turned in with the exam.

Points breakdown:

Question	1	2	3	4	5	6	7	Total
Points	14	5	15	16	20	5	25	100

Useful formulae:

Shortcut formula for variance of RV X: $\sigma_X^2 = E[X^2] - (E[X])^2$

If X is Bernoulli RV with prob. of success π, then $P(X = 1) = \pi$ and $P(X = 0) = 1 - \pi$.

If X is Binomial with prob. of success π for n independent Bernoulli trials, then $f(x) = C_x^n \pi^x (1 - \pi)^{n-x}$

If X is Poisson with expected successes per interval λ then $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, where $e = 2.718...$

1. (14 points) The below table lists the distance from campus and commuting time for a sample of five Western students:

Student	Distance from campus (km)	Commuting time (minutes)	Transportation mode
1	2	30	walk
2	3	15	bike
3	0.5	7	walk
4	10	15	car
5	4	20	bike

   (a) (4 points) What is the median distance and median commuting time in the sample?

   (b) (5 points) What is the mean distance and mean commuting time in the sample?

   (c) (5 points) What is the sample covariance between distance and commuting time?

2. (5 points) There are 50 men and 600 women spelunking in a cave. If 40 people are randomly hugged by a cave bear, what is the probability that at least two of those hugged are men?

3. (15 points) Let X be a discrete random variable which can take the values $\{\frac{1}{3}, \frac{2}{3}, 1\}$, with the cumulative distribution function $F(x_0) = x_0^3$.

   (a) (5 points) What is the probability mass function for X, $P(x)$? I want numbers here, not an algebraic expression.

   (b) (5 points) Calculate σ_X.

   (c) (5 points) Say X represents a stock price, and Sally the trader get a bonus b which depends on the stock price according to $b(X) = 10,000X^3$. Sally's utility u is a function of her total income, Y, which is her base salary of 90,000 plus any bonus she earns, according to $u(Y) = Y^{1/2}$. What is Sally's expected utility?

4. (20 points) Let X be a Poisson random variable representing the number of buses arriving in the time interval of interest (these are specified below). If there is no bus driver strike, we expect 8 buses to arrive per hour. If there is a bus driver strike, we expect no buses to arrive per hour.

   You are trying to remember whether a strike had been called, so you can figure out whether you should just walk the two blocks to class. Your prior belief was that there was a 25% chance there was a strike today, but you forgot to check the newspaper before leaving to see whether there was a strike.

   (a) (5 points) What is your posterior belief that there is a strike if no buses arrive in the next 10 minutes?

   (b) (5 points) What is your posterior belief that there is a strike if one bus arrives in the next 10 minutes?

   (c) (10 points) Express, as a function of time measured in minutes, t, your posterior probability that there is a strike if there have been no buses for the past t minutes. Is this function increasing or decreasing in t, and give some intuition why.

5. (16 points) These tables represent the probabilities of events B and C, which depend on whether or not event A has occurred. $P(A) = 0.25$.

	Given A	
	C	$\overline{C}$
B	0.08	0.72
$\overline{B}$	0.02	0.18

	Given $\overline{A}$	
	C	$\overline{C}$
B	0.28	0.12
$\overline{B}$	0.42	0.18

   (a) (4 points) What is the probability of C given that A has occurred?

   (b) (4 points) What is the probability of event C?

   (c) (4 points) Are B and $\overline{B}$ independent events?

   (d) (4 points) Are B and C independent events?

6. (5 points) Let X and Y be jointly distributed discrete random variables denoting the price of a share of stock X or stock Y, respectively. Suppose that you know $\mu_X > \mu_Y$ and $\sigma_X^2 > \sigma_Y^2$.

   If $E[Y|X = 10] < E[Y|X = 5]$, is the correlation in share prices, ρ_{XY}, positive, negative, zero, or do you not have enough information to say? Why? You can provide a reason without providing a mathematical proof, if you'd prefer.

7. (25 points) Let X, Y, and W be discrete random variables with the following marginal probability mass functions:

Value	$P_X(x)$	$P_Y(y)$	$P_W(w)$
1	0.5	b	0.5
10	0.5	$1-b$	0
c	0	0	0.5

where the constants $b < 0.5$ and $c > 10$. Denote the means of X, Y, and W as μ_X, μ_Y, and μ_W, and their variances as σ_X^2, σ_Y^2, and σ_W^2.

(a) (5 points) Is μ_X greater than, less than, or equal to μ_Y, or do you not have enough information to say? Why?

(b) (5 points) Is μ_Y greater than, less than, or equal to μ_W, or do you not have enough information to say? Why?

(c) (5 points) Is σ_X^2 greater than, less than, or equal to σ_Y^2, or do you not have enough information to say? Why?

(d) (5 points) Is σ_X^2 greater than, less than, or equal to σ_W^2, or do you not have enough information to say? Why?

(e) (5 points) Is σ_Y^2 greater than, less than, or equal to σ_W^2, or do you not have enough information to say? Why?