Université d’Ottawa
Faculté de genie

Départment de
Génie Méchanique

uOttawa
L’Université canadienne
Canada’s university

University of Ottawa
Faculty of Engineering

Department of
Mechanical Engineering

MCG 4303
Mechanical Vibration Analysis

FINAL EXAMINATION

Length of Examination: 3 hours **April 15, 2009**

Professor: Natalie Baddour **Page 1 of 5**

Family Name: ____

Other Names: ____

Student Number: ___________

Signature __________________________

Closed book.

If you do not understand a question, clearly state an assumption and proceed.

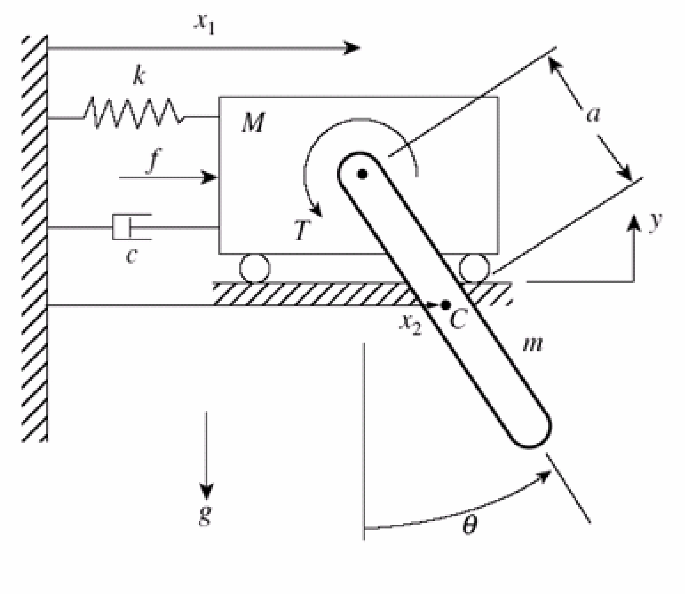
There are **FIVE** questions. All questions carry equal weight.

At the end of the exam, when time is up:
- Stop working and turn your exam upside down.
- Remain silent.
- Do not move or speak until all exams have been picked up, and a TA or the Professor gives the go-ahead to leave.

Question 1

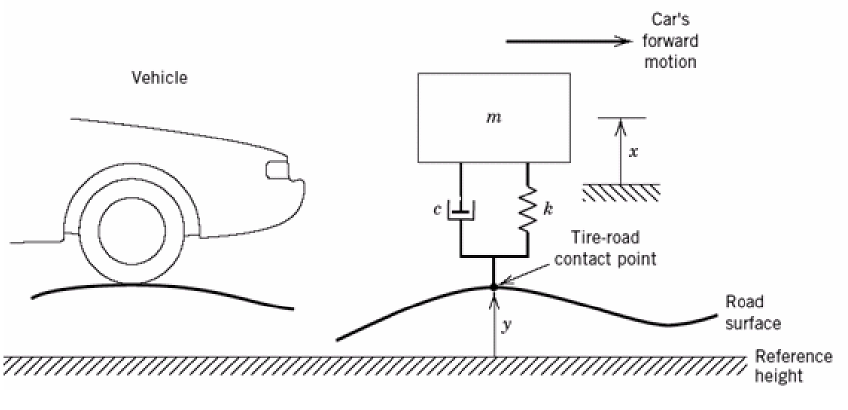
Consider the 2 DoF system shown in the figure. The mass of the cart is M and the moment of inertia of the bar about its centre of mass is I_G. The mass of the bar is m. x_1 is the displacement of the cart and the angle θ is the angular displacement of the car from its equilibrium position. A moment applied on the bar where it is pinned to the cart is given by T and f is a linear force on the cart.

(a) Write the kinetic and potential energies of the system as well as the Raleigh Dissipation function.
(b) Use Lagrange's equations to derive the equations of motion for the cart-bar system.
(c) Using the small angle approximation for θ, find the linearized equations of motion.
(d) Write the linearized equations of motion in matrix form.



Question 2

Consider the single degree-of-freedom quarter-car suspension model as shown in the figure.



The equation of motion for this system is given by

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

The vehicle parameters are $m=2$ kg, $c=4$ Ns/m, $k=4$ N/m. The problem you are trying to model is one of a suspension that is undisturbed until it hits a curb of height 1/10 m at t=5 s.

(a) How would you choose $y(t)$ in order to model this problem as stated above?
(b) Write down the full equation of motion to describe this problem, including initial conditions as well as y and $\dot{y}$.
(c) Find the total response of the vehicle chassis (mass *m*) to this problem.
(d) Draw a rough sketch of what you expect the vehicle chassis response of part (c) to look like.

Question 3

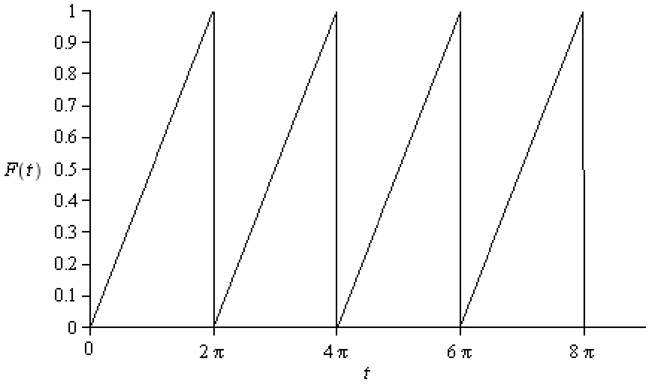
The equation of motion of a single degree-of-freedom system can be written as

$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2x=F(t)$$

(a) Find the frequency response of this system (magnitude and phase).
(b) Draw a rough sketch of what you expect the magnitude of the frequency response to look like for a lightly damped system.
(c) Find the system response to the sawtooth function shown in the figure and given by the Fourier series:

$$F(t)=\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{-1}{n\pi}\right)\sin\left(nt\right)$$

(d) Is the response you found in part (c) the total response of the system? Why or why not?



Question 4

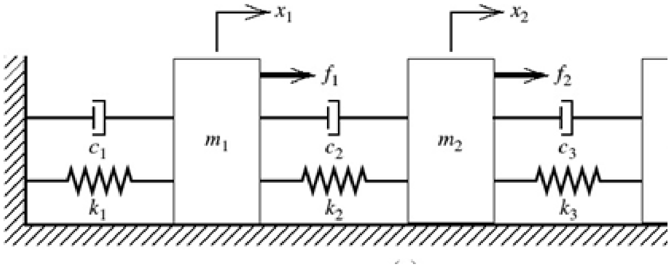
The equation of motion of a 2 degree of freedom system is given by

$$\begin{bmatrix}2 & 1\\ 1 & 5\end{bmatrix}\ddot{\vec{x}}+\begin{bmatrix}3 & 0\\ 0 & 6\end{bmatrix}\vec{x}=\vec{0}.$$

(a) Find the natural frequencies and modeshapes.
(b) Sketch the modeshapes for this system.
(c) Write down the general form of the free vibration response.
(d) What do you expect the equation of motion in modal coordinates to be?
(e) Find the mass-normalized modal matrix for this system.

Question 5

Consider the following damped 2 degree of freedom system shown in the figure:



The equation of motion for this system is given by

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \ddot{\vec{x}} + \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \dot{\vec{x}} + \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \vec{x} = \begin{bmatrix} -2\cos(t) \\ 2\cos(t) \end{bmatrix}$$

All initial conditions are assumed to be zero (positions and velocities of both masses). The modal matrix for this problem is $P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

(a) Is this an example of proportional damping or not?
(b) Using the given modal matrix, find the equation of motion in modal coordinates.
(c) What are the natural frequencies *and* damped frequencies of each mode?
(d) Do you expect the given system to exhibit resonance? Explain your answer.

Useful formulae

Roots of $ax^2+bx+c:\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\int udv = uv - \int vdu$$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$\sin(u) = \frac{e^{iu}-e^{-iu}}{2i} = \mathrm{Im}\left\{e^{iu}\right\}$$

$$\cos(u) = \frac{e^{iu}+e^{-iu}}{2} = \mathrm{Re}\left\{e^{iu}\right\}$$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos(n\omega t) + b_n\sin(n\omega t)\right]$$

Fourier Coefficients:

$$a_0 = \frac{2}{T}\int_0^T x(t)dt = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} x(t)dt$$

$$a_n = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} x(t)\cos(n\omega t)dt$$

$$b_n = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} x(t)\sin(n\omega t)dt$$

If $\mathrm{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\mathrm{A}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial R}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

Small angle approximation:

$\sin(\theta)\approx\theta \qquad \cos(\theta)\approx 1$

Or $\cos(\theta)\approx 1-\dfrac{\theta^2}{2}$

Laplace Transforms

$f(t)$	$F(s) = L\{f(t)\}$
1	$\frac{1}{s}$
$\delta(t)$	1
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
e^{at}	$\frac{1}{s-a}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$u(t-a)$	$\frac{1}{s}e^{-as}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$