# CVG2140 – Solutions to Assignment No. 8 (Combined Stresses)

**Problem 1.** Determine the principal stresses and their directions at the surface point C of the solid cylinder shown in Fig. 1 (diameter = 50 mm, length = 250 mm) subjected to an axial force $P$ and torque $T$.

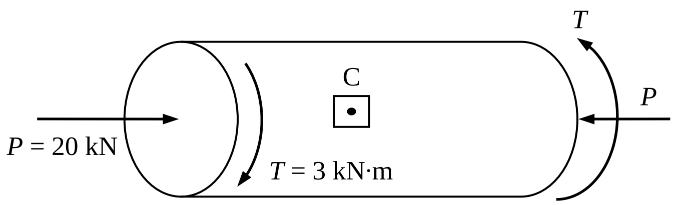



Fig. 1

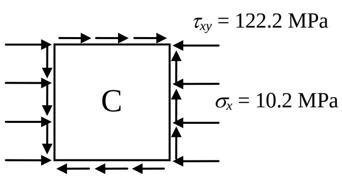
At C:

$$\sigma_x = \frac{P}{A} = \frac{-20\times 10^3}{\pi \times 25^2} = \text{-10.2 MPa}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16\times 3\times 10^6}{\pi \times 50^3} = 122.2 \text{ MPa}$$

The state of stress at point C is therefore:




$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times 122.2}{-10.2} = -24 \quad \Rightarrow \quad 2\theta_p = -87.6^\circ \quad \Rightarrow \quad \theta_p = \begin{cases} -43.8^\circ \\ -43.2^\circ + 90^\circ = 46.2^\circ \end{cases}$$

Substituting $\theta_p$ and $\theta_p$+90° into the transformation equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{-10.2}{2} + \frac{-10.2}{2}\cos(-87.6) + 122.2\sin(-87.6) = -127.4 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{-10.2}{2} + \frac{-10.2}{2}\cos(2\times 46.2) + 122.2\sin(2\times 46.2) = 117.2 \text{ MPa}$$

Therefore, $\sigma_1 = \sigma_{max}$ = 117.2 MPa at 46.2° and $\sigma_2 = \sigma_{min}$ = -127.4 MPa at -43.8°

**Problem 2.** Determine the principal stresses and their directions at three surface locations, *A*, *B*, and *C*, of the brass cantilever beam with rectangular cross-section shown in Fig. 2. The beam is subjected to its own weight as well as a concentrated load of 12 kN at its free end. The material properties of brass are as follows: density $\rho$ = 8500 kg/m$^3$, Young's modulus $E$ = 100 GPa, and yield stress $\sigma_y$ = 450 MPa.

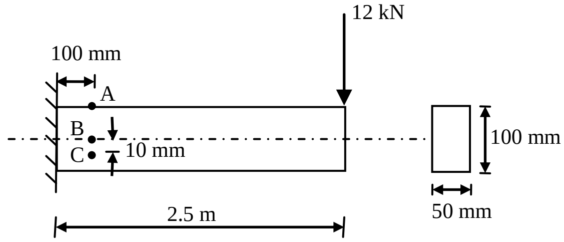
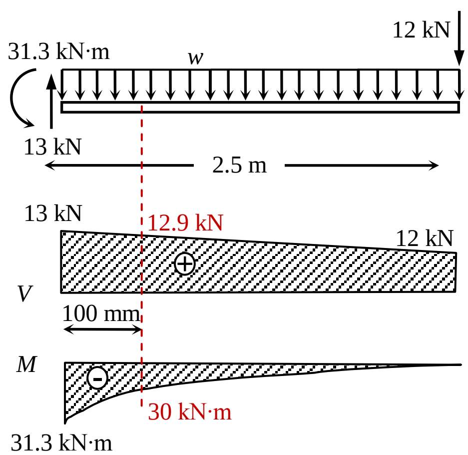



Fig. 2

The weight per linear meter of the cantilever beam is:

$$w = \rho g A = 8500 \times 9.81 \times 0.1 \times 0.05 = 0.417 \text{ kN/m}$$




At the three locations:

$$\sigma_x = -\frac{My}{I} = -\frac{12My}{bh^3}, \quad \sigma_y = 0, \quad \tau_{xy} = \frac{VQ}{Ib} = \frac{12VQ}{b^2h^3}$$

| | $Q$ (mm$^3$) | $\sigma_x$ (MPa) | $\tau_{xy}$ (MPa) | $\sigma_1$ (MPa) | $\theta_{p1}$ | $\sigma_2$ (MPa) | $\theta_{p2}$ |
|---|---|---|---|---|---|---|---|
| **A** | 0 | 360 | 0 | 360 | 0° | 0 | 90° |
| **B** | $62.5\times10^3$ | 0 | -3.87 | 3.87 | 45° | -3.87 | 135° |
| **C** | $60\times10^3$ | -72 | -3.71 | 0.19 | 92.9° | -72.19 | 2.9° |

**Problem 3.** The state of stress at a point is shown in Fig. 3. By using Mohr's circle, determine:

a) The principal stresses and their orientation;

b) The maximum in-plane shear stress and average normal stress, specifying the corresponding orientation; and,

c) The equivalent state of stress if the coordinate system is oriented 30º clockwise from the element shown in Fig. 3.

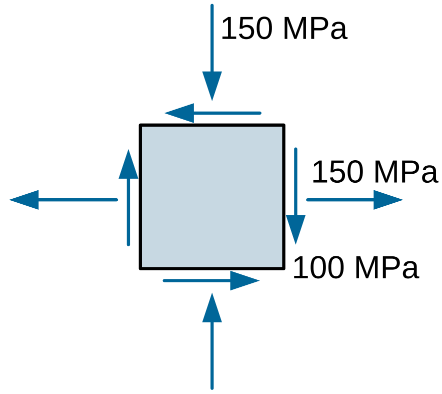



Fig. 3

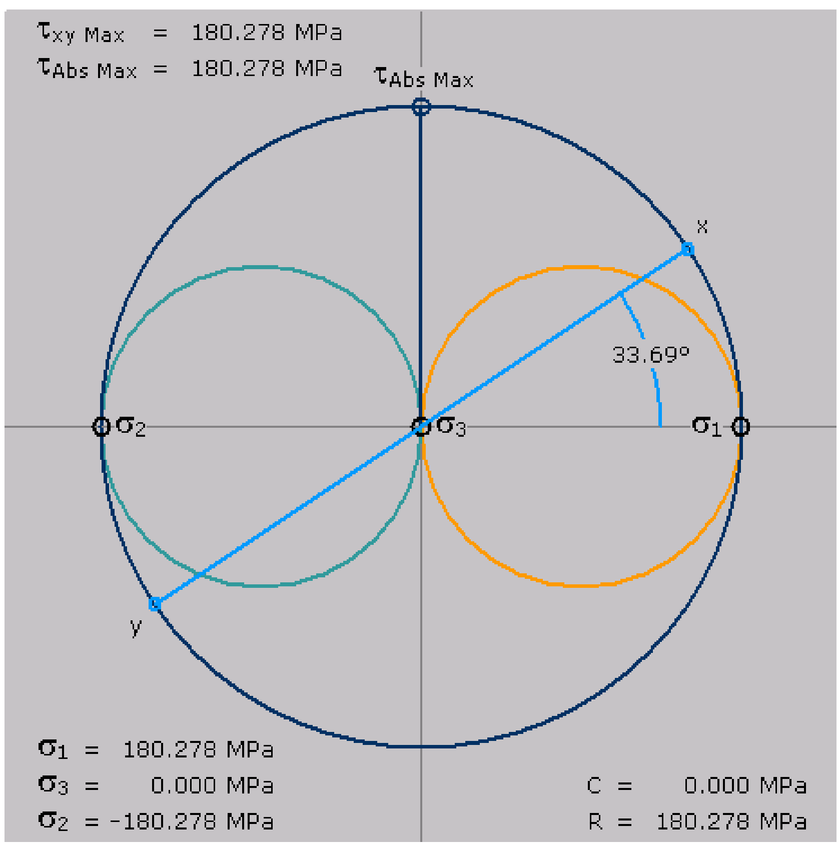
From Fig. 3:

$\sigma_x$ = 150 MPa, $\sigma_y$ = -150 MPa, $\tau_{xy}$ = -100 MPa

Therefore,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{150 - 150}{2} = 0 \text{ MPa}$$

$$R = \tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{150 + 150}{2}\right)^2 + 100^2} = 180.28 \text{ MPa}$$

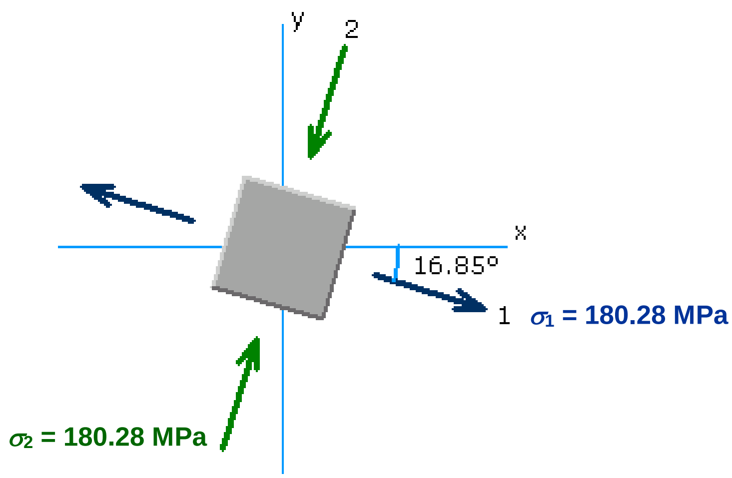


a) $2\theta_p = \tan^{-1}\left(\frac{100}{150}\right) = 33.69^\circ$

$\theta_p$ = -33.69°/2 = -16.85° (negative, because the rotation from $x$ to $\sigma_1$ is clockwise)

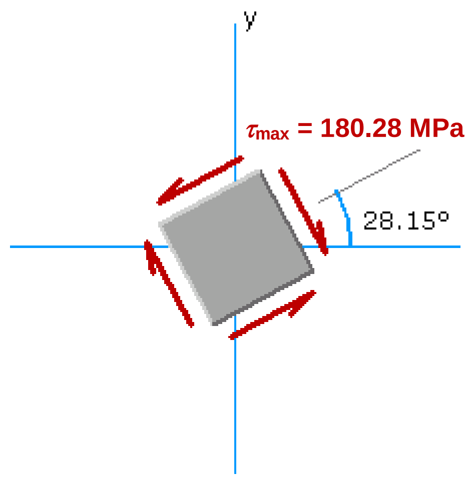
$\sigma_1$ = 0 + 180.28 = 180.28 MPa acting at -16.85°

$\sigma_2$ = 0 – 180.28 = -180.28 MPa acting at -16.85°+90°=73.15°

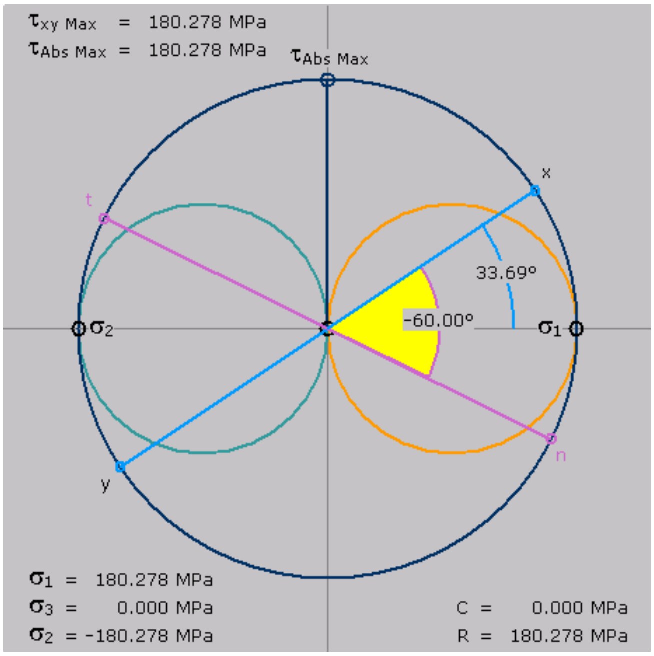


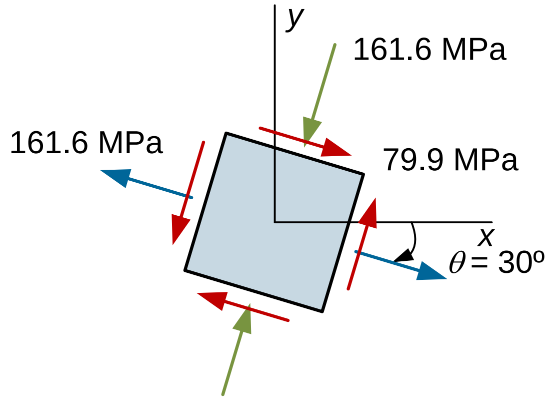

b) $\tau_{max}$ = R = 180.28 MPa acting at -16.85°+ 45° = 28.15°

$\sigma_{avg}$ = C = 0 MPa



c)




$\sigma_{x'}$ = R×cos(60°-33.69°) = 180.28×cos(26.31°)= 161.6 MPa

$\sigma_{y'}$ = - R×cos(60°-33.69°) = -161.6 MPa

$\tau_{x'y'}$ = R×sin(60°-33.69°) = 180.28×sin(26.31°) = 79.9 MPa