CVG2140 – Solutions to Assignment No. 7 (Shear & Torsion)

Problem 1. The wood beam shown in Fig. 1 has an allowable shear stress of τ_{allow} = 7 MPa. Determine the maximum shear force V that can be applied to the cross section.

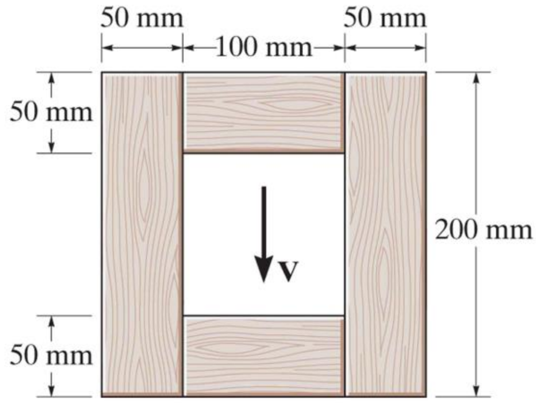


Fig. 1

$$I = \frac{200\times200^3}{12} - \frac{100\times100^3}{12} = 125\times10^6 \text{ mm}^4$$

$$\tau_{allow} = \frac{VQ_{\max}}{Ib} \quad \Rightarrow \quad V = \frac{\tau_{allow}Ib}{Q_{\max}} = \frac{7\times125\times10^6\times100}{(200\times50\times75)+(2\times50\times50\times25)} = \underline{100\text{ kN}}$$

Problem 2. Determine the absolute maximum shear stress in the T-beam shown in Fig. 2, as well as the maximum shear stress in the cross section that passes through point C.

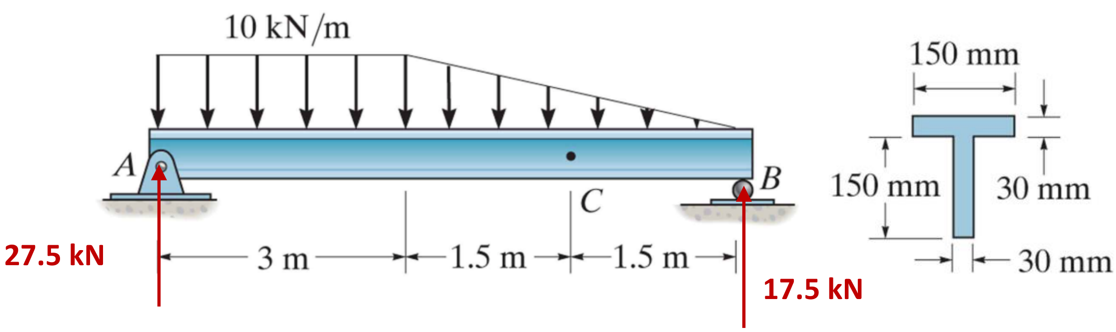
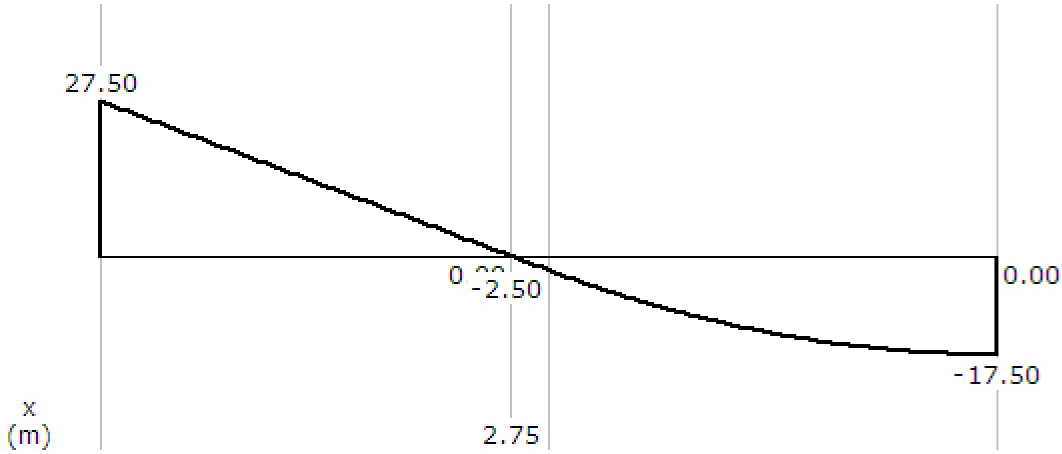
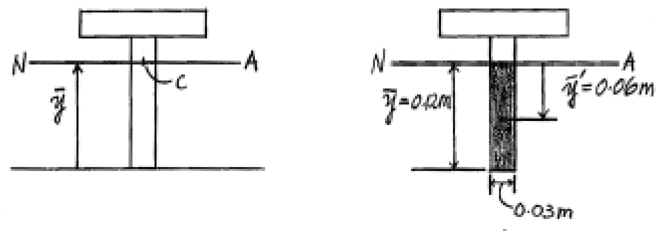


Fig. 2

The shear force diagram, as shown below, indicates that $V_{\max}$ = 27.50 kN at support A.



The neutral axis passes through the centroid of the cross section as shown in the figure:



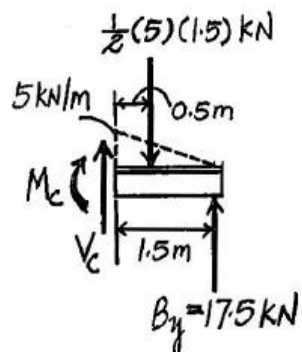
$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(30\times150\times75)+(150\times30\times165)}{(30\times150)+(150\times30)} = 120 \text{ mm (from the bottom)}$$

$$I_{NA} = \frac{150\times30^3}{12} + \left(150\times30\times45^2\right) + \frac{30\times150^3}{12} + \left(150\times30\times45^2\right) = 27\times10^6 \text{ mm}^4$$

The absolute maximum shear stress occurs at the neutral axis where Q is maximum, i.e.,

$$\tau_{\max} = \frac{VQ_{\max}}{Ib} = \frac{27.5\times10^3\times(120\times30\times60)}{27\times10^6\times30} = \underline{7.33 \text{ MPa}}$$

The shear force at C is determined from the following FBD:



$$\sum F_y = 0; \quad V_c + 17.5 - \frac{5\times1.5}{2} = 0 \quad \Rightarrow \quad V_c = \text{-}13.75 \text{ kN}$$

The maximum shear stress at section C is therefore given by:

$$\tau_{\max}^{C} = \frac{VQ_{\max}}{Ib} = \frac{13.75\times10^3\times(120\times30\times60)}{27\times10^6\times30} = \underline{3.67 \text{ MPa}}$$

Problem 3. The 60-mm-diameter solid shaft in Fig. 3 is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the outer surface of the shaft and specify their locations, measured from the fixed end *A*.

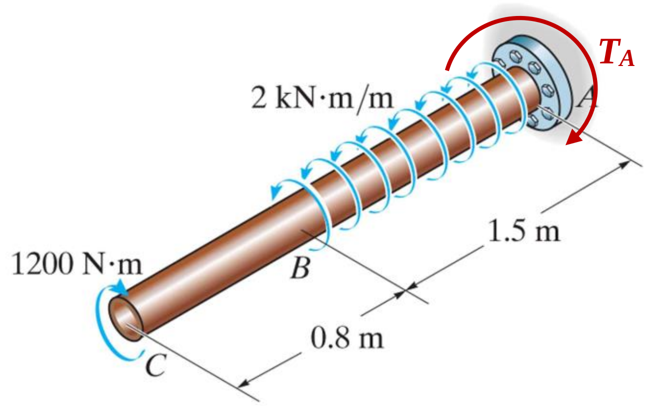
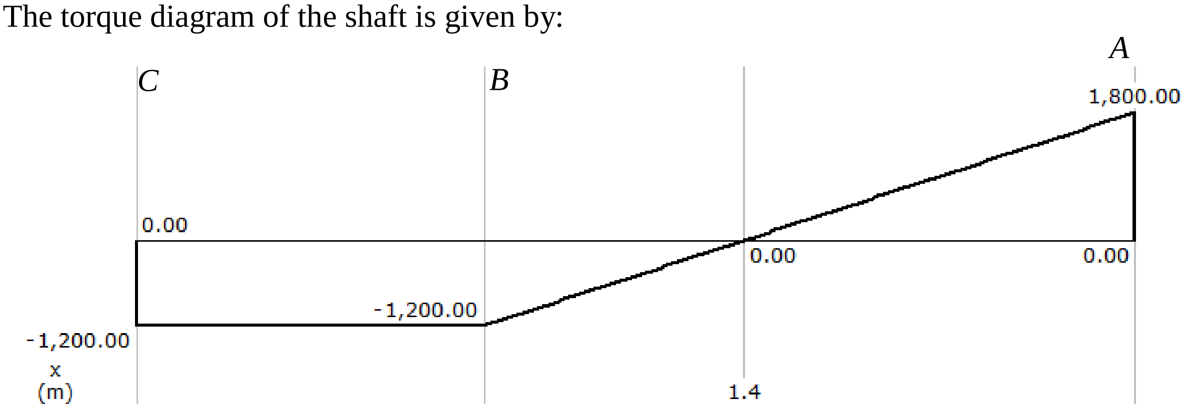


Fig. 3

By applying equilibrium:

$$\sum T = 1200 - (2000 \times 1.5) + T_A = 0 \quad \Rightarrow \quad T_A = 1800 \text{ N} \cdot \text{m}$$

The torque diagram of the shaft is given by:



The minimum shear stress occurs when the internal torque is zero in segment *AB*. By setting $T_{AB} = 0$,

$$0 = (2000x - 1200) \text{ N} \cdot \text{m} \quad \Rightarrow \quad x = 0.6 \text{ m}$$

Therefore, $\tau_{\min} = 0$ at a distance from *A* of 1.5 m – 0.6 m = 0.9 m.

The maximum shear stress occurs at *A* where the torque is the greatest, i.e.,

$$\tau_{\max} = \tau_A = \frac{T_A r}{J} = \frac{1800 \times 10^3 \times 30}{\frac{\pi}{2} \times 30^4} = \underline{42.4 \text{ MPa}}$$

Problem 4. The A-36 steel assembly in Fig. 4 consists of a tube having an outer radius of 1 in and a wall thickness of 0.125 in. Using a rigid plate at *B*, it is connected to the solid 1-in-diameter shaft *AB*. Determine the rotation of the tube's end *C* if a torque of 200 lb·in is applied to the tube at this end. The end *A* of the shaft is fixed. $G = 11\times10^6$ psi.

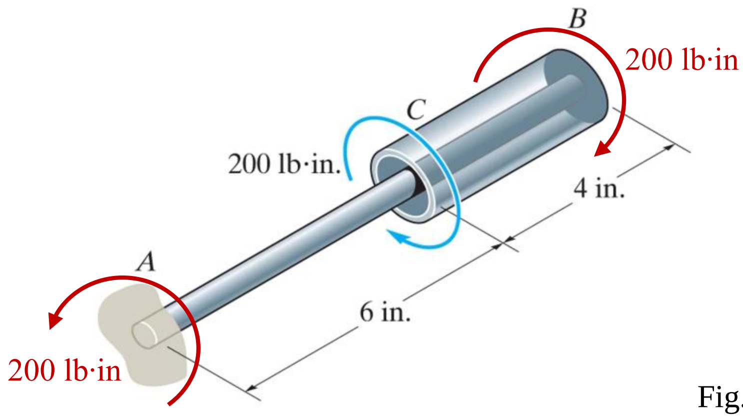


Fig. 4

$$\phi_B = \frac{T_{AB}L}{GJ} = \frac{200\times10}{11\times10^6\times\frac{\pi}{2}\times0.5^4} = 0.001852 \text{ rad}$$

$$\phi_{C/B} = \frac{T_{CB}L}{GJ} = \frac{-200\times4}{11\times10^6\times\frac{\pi}{2}\times\left(1^4-0.875^4\right)} = \text{-}0.0001119 \text{ rad}$$

$$\phi_C = \phi_B - \phi_{C/B} = 0.001852 + 0.0001119 = \underline{0.001964 \text{ rad} = 0.113°}$$