

CVG2140 – Solutions to Assignment No. 5 (Flexural Stresses)

Problem 1. A simply supported wood beam AB with span length $L = 3.5$ m carries a uniform load of intensity $q = 6.4$ kN/m (see Fig. 1). Calculate the maximum bending stress σ_{\max} due to the load q if the beam has a rectangular cross section with width $b = 150$ mm and height $h = 280$ mm.

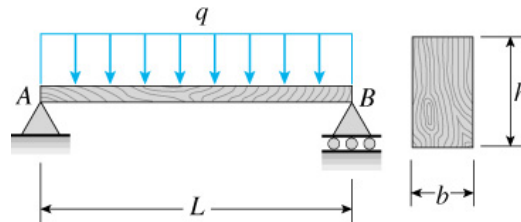


Fig. 1

For a simple-supported beam under the action of uniformly distributed load q , the maximum bending moment occurs at mid-span, and its value is:

$$M_{\max} = \frac{ql^2}{8}, \text{ where } l \text{ is the span length}$$

$$\sigma_{\max} = \frac{M_{\max} \times y_{\max}}{I} = \frac{M_{\max}}{S}, \quad S = \frac{bh^2}{6} \Rightarrow \sigma_{\max} = \frac{6M_{\max}}{bh^2} = \frac{3ql^2}{4bh^2}$$

Substituting numerical values results in:

$$\sigma_{\max} = \frac{3 \times 6.4 \times (3.5 \times 10^3)^2}{4 \times 150 \times 280^2} = \underline{5.0 \text{ MPa}}$$

Problem 2. A seesaw weighing 5 kg/m of length is occupied by two children, each weighing 40 kg (see Fig. 2). The centre of gravity of each child is 2.5 m from the fulcrum. The board is 5.8-m long, 20-cm wide and 4-cm thick. What is the maximum bending stress in the board?

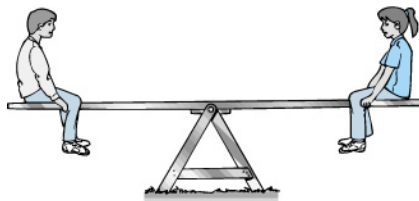
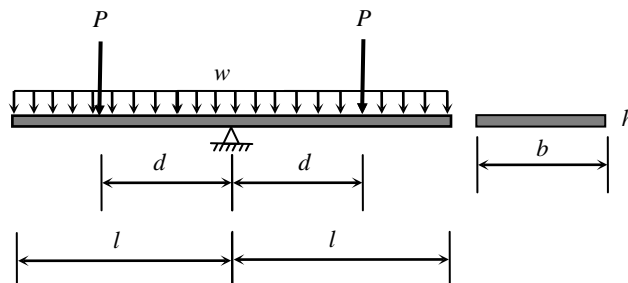


Fig. 2



$$w = 5 \text{ kg/m} \times 9.8 \text{ m/s}^2 = 49 \text{ N/m}, P = 40 \text{ kg} \times 9.8 \text{ m/s}^2 = 392 \text{ N}, d = 2.5 \text{ m}, l = 2.9 \text{ m},$$

$$b = 20 \text{ cm}, h = 4 \text{ cm}$$

The maximum bending moment occurs at mid-span, and its value is:

$$M_{\max} = Pd + \frac{wl^2}{2} = (392 \times 2.5) + \left(\frac{49 \times 2.9^2}{2} \right) = 1,186.1 \text{ N}\cdot\text{m}$$

$$\sigma_{\max} = \frac{M_{\max}}{S}, \quad S = \frac{bh^2}{6} \Rightarrow \sigma_{\max} = \frac{6M_{\max}}{bh^2} = \frac{6 \times 1186.1}{0.2 \times 0.04^2} = \underline{\underline{22.2 \text{ MPa}}}$$

Problem 3. A small dam of height $h = 2.4 \text{ m}$ is constructed of vertical wood beams AB of thickness $t = 160 \text{ mm}$, as shown in Fig. 3. Consider the beams to be simply supported at the top and bottom. Determine the maximum bending stress σ_{\max} in the beams, assuming that the weight density of water is $\gamma = 9.81 \text{ kN/m}^3$. Hint: the hydrostatic pressure p in still water is $p = \gamma d$ (Pa), where γ is the weight density of water (N/m^3) and d is the depth from the free surface (m). Do the problem for 1 m of the dam width.

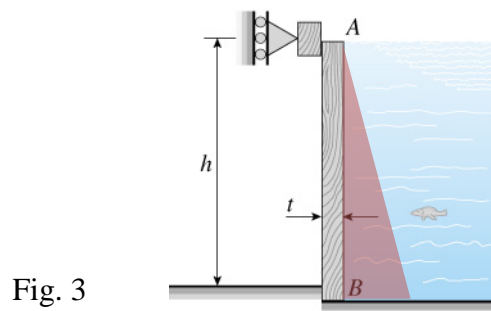
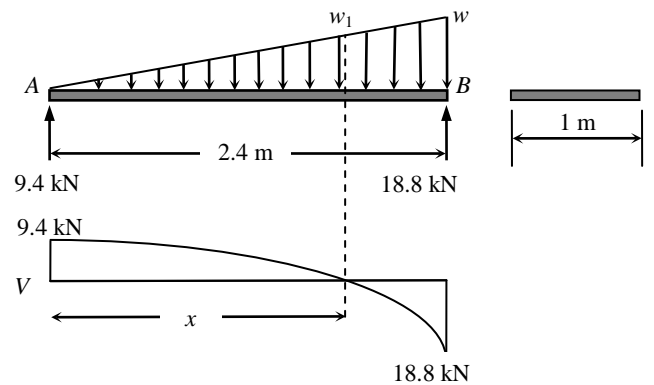
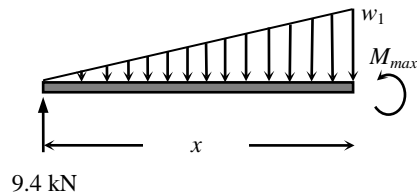


Fig. 3

$$w = \gamma \times h \times \text{width} = 9.81 \times 2.4 \times 1 = 23.5 \text{ kN/m}$$



The maximum moment occurs at x , since $V = \frac{dM}{dx} = 0$. By establishing equilibrium at that section:



$$\sum F_y = 9.4 - \left(\frac{1}{2} \times w_1 \times x \right) = 0$$

$$\text{Since } \frac{w_1}{x} = \frac{w}{2.4}, \quad 9.4 - \left(\frac{1}{2} \times \frac{23.5}{2.4} \times x^2 \right) = 0 \Rightarrow x = 1.39 \text{ m}$$

The moment at $x = 1.39 \text{ m}$ is then calculated by establishing moment equilibrium at that section:

$$\sum M = -(9.4 \times 1.39) + \left(\frac{1}{2} \times 13.6 \times 1.39 \times \frac{1}{3} \times 1.39 \right) + M_{\max} = 0 \Rightarrow M_{\max} = 8.69 \text{ kN}\cdot\text{m}$$

$$\sigma_{\max} = \frac{M_{\max}}{S}, \quad S = \frac{bt^2}{6} \Rightarrow \sigma_{\max} = \frac{6M_{\max}}{bt^2} = \frac{6 \times 8.69 \times 10^3}{1 \times 0.16^2} = \underline{2 \text{ MPa}}$$

Problem 4. The beam shown in Fig. 4 has a T cross section with moments of inertia with respect to y and z equal to $I_y = 15.63 \times 10^6 \text{ mm}^4$ and $I_z = 53.13 \times 10^6 \text{ mm}^4$, respectively. For the beam section where the absolute value of the internal bending moment is maximum, $M = |M_{\max}|$, find the maximum bending stress $|\sigma_{\max}|$ and the bending stress @ H σ_H (is σ_H compressive or tensile?). Considering the entire beam, find the maximum tensile bending stress $\sigma_{\max, \text{Tensile}}$.

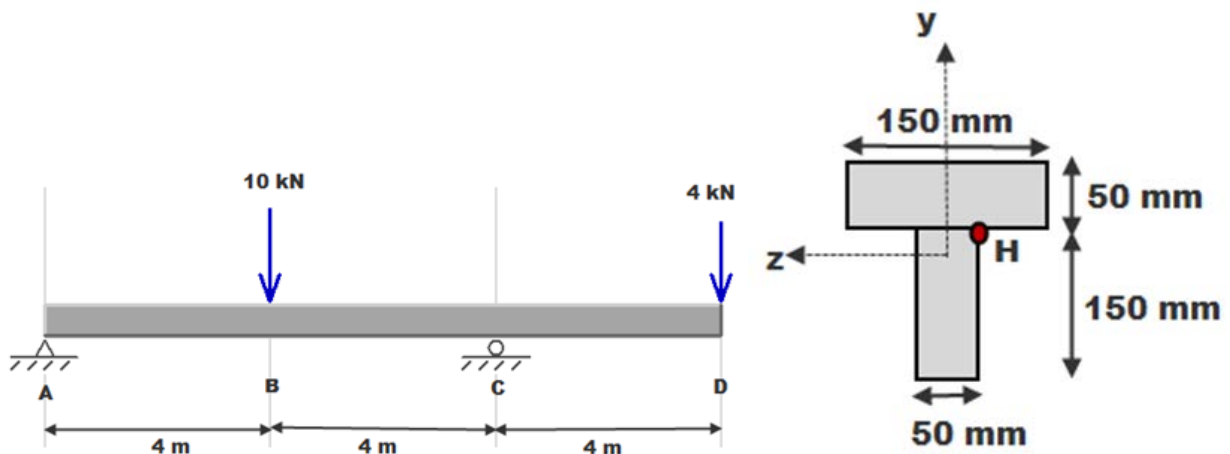
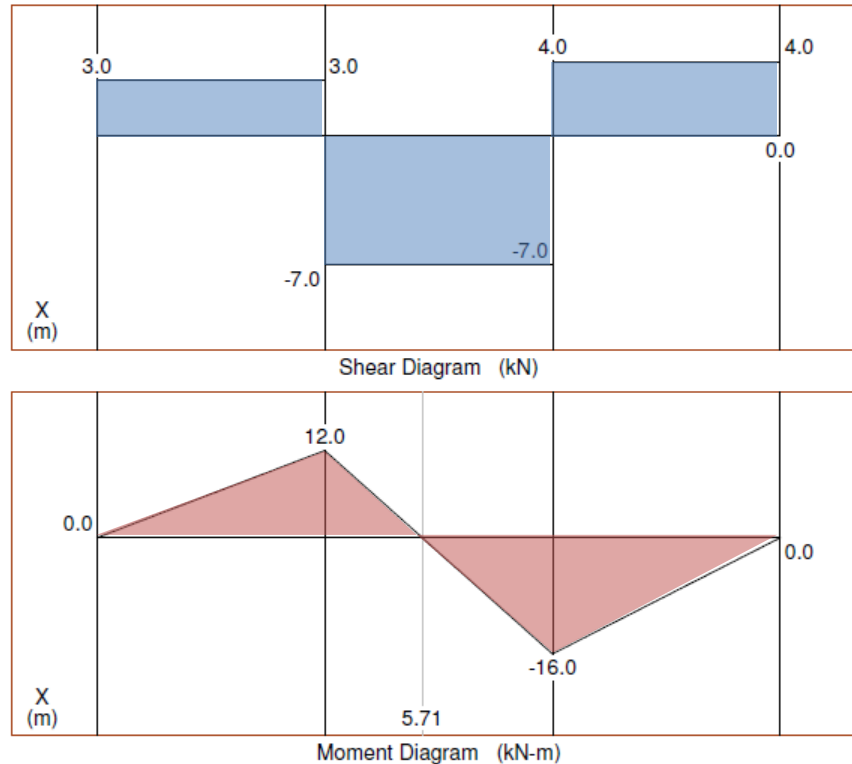


Fig. 4

From equilibrium:

$$\left. \begin{aligned} \sum F_x &= A_x = 0 \\ \sum F_y &= A_y + C_y - 14 = 0 \\ \sum M_A &= -(10)(4) + (C_y)(8) - (4)(12) = 0 \end{aligned} \right\} \Rightarrow \begin{cases} A_y = 3 \text{ kN} \\ C_y = 11 \text{ kN} \end{cases}$$

The shear and bending moment diagrams of the beam are as follows:



a) σ_{\max} from the BMD, $|M_{\max}| = 16 \text{ kN}\cdot\text{m}$

(i) The neutral axis (or centroid) of the cross section is located at:

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{(50 \times 150 \times 175) + (50 \times 150 \times 75)}{(50 \times 150) + (50 \times 150)} = 125 \text{ mm from the bottom}$$

(ii) The maximum bending stress $|\sigma_{\max}|$ occurs at the bottom fibre where $c = 125 \text{ mm}$.

$$|\sigma_{\max}| = \left| \frac{M_{\max} c}{I_z} \right| = \frac{(16 \times 10^6)(125)}{(53.125 \times 10^6)} = 37.6 \text{ MPa}$$

(Note that this is a compressive stress.)

(iii) The bending stress σ_H :

$$\sigma_H = -\frac{M_{\max} y_H}{I_z} = -\frac{(-16 \times 10^6)(25)}{(53.125 \times 10^6)} = 7.53 \text{ MPa (tensile stress)}$$

b) The maximum tensile bending stress $\sigma_{\max, \text{ tensile}}$:

Considering the section where $M = M_{\max}^- = -16 \text{ kN}\cdot\text{m}$, $\sigma_{\max, \text{ tensile}}$ will develop at the top fibre where $c = 75 \text{ mm}$.

$$\sigma_{\max, \text{ tensile}} = -\frac{M_{\max} c}{I_z} = -\frac{(-16 \times 10^6)(75)}{(53.125 \times 10^6)} = 22.6 \text{ MPa}$$

Considering the section where $M = M_{\max}^+ = 12 \text{ kN}\cdot\text{m}$, $\sigma_{\max, \text{ tensile}}$ will develop at the bottom fibre where $c = -125 \text{ mm}$.

$$\sigma_{\max, \text{ tensile}} = -\frac{M_{\max} c}{I_z} = -\frac{(12 \times 10^6)(-125)}{(53.125 \times 10^6)} = \underline{28.2 \text{ MPa}}$$

Therefore, the maximum tensile bending stress $\sigma_{\max, \text{ tensile}}$ develops at the bottom fibre of the section with maximum positive moment, and its value is $\underline{\sigma_{\max, \text{ tensile}} = 28.2 \text{ MPa}}$.