CVG2140 – Solution to Assignment No. 4 (Axial Members)

Problem 1. The A-36 steel rod (*E* = 200 GPa) is subjected to the loading shown in Fig. 1. If the cross-sectional area of the rod is 50 mm^2, determine the displacement of its end *D*. Neglect the size of the couplings at *B*, *C*, and *D*.

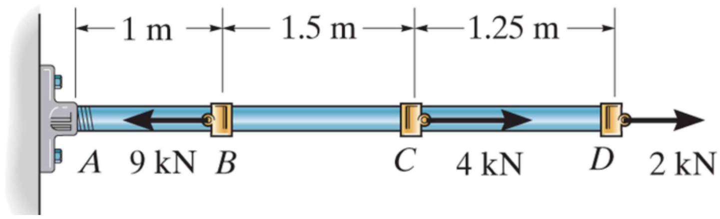


Fig. 1

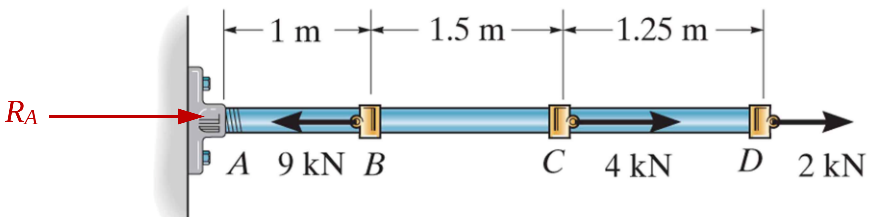
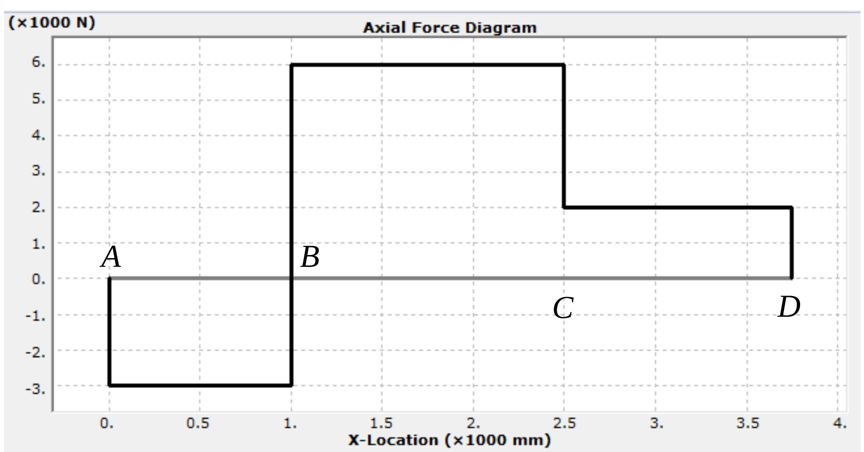


Fig. 1

By establishing equilibrium:

$$\sum F_x = R_A - 9 + 4 + 2 = 0 \quad \Rightarrow \quad R_A = 3\text{ kN}$$

The distribution of the internal axial load *N*(*x*) along the steel rod is shown below:



The displacement of end *D* is therefore given by:

$$\delta_D = \delta_{B/A} + \delta_{C/B} + \delta_{D/C} = \frac{F_{BA}L_{BA}}{EA} + \frac{F_{CB}L_{CB}}{EA} + \frac{F_{DC}L_{DC}}{EA}$$

$$\delta_D = -\frac{3\times10^3\times1,000}{200\times10^3\times50} + \frac{6\times10^3\times1,500}{200\times10^3\times50} + \frac{2\times10^3\times1,250}{200\times10^3\times50} = \underline{0.85\text{ mm}}$$

Problem 2. The assembly consists of two A-36 steel rods and a rigid bar BD. Each rod has a diameter of 0.75 in. If a force of 10 kip is applied to the bar as shown in Fig. 2, determine the vertical displacement of the load. E_{st} = 29,000 ksi.

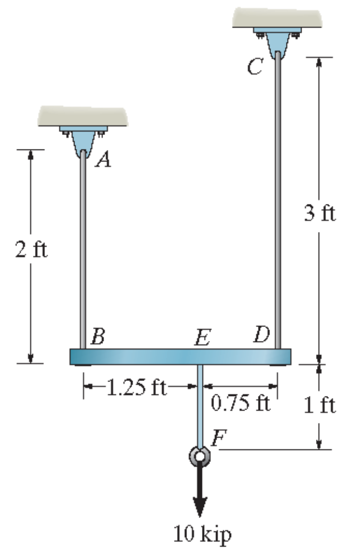


Fig. 2

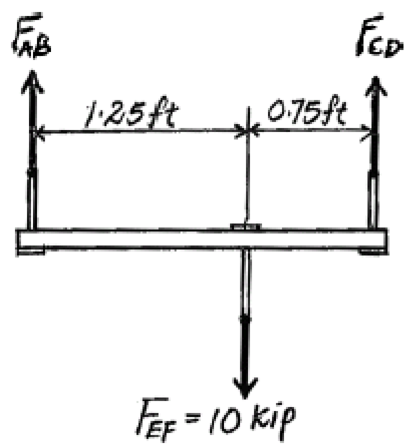


Fig. 2(a)

From the FBD shown in Fig. 2(a):

$$\sum M_B = (F_{CD})(2) - (10)(1.25) = 0 \quad \Rightarrow \quad F_{CD} = 6.25 \text{ kip}$$

$$\sum M_D = -(F_{AB})(2) + (10)(0.75) = 0 \quad \Rightarrow \quad F_{AB} = 3.75 \text{ kip}$$

The elongations of bars *AB* and *CD* are given by:

$$\delta_B = \frac{F_{AB} L_{AB}}{E_{st} A} = \frac{3.75 \times 2 \times 12}{29 \times 10^3 \times \pi \times 0.75^2 / 4} = 0.007025 \text{ in } \downarrow$$

$$\delta_D = \frac{F_{CD} L_{CD}}{E_{st} A} = \frac{6.25 \times 3 \times 12}{29 \times 10^3 \times \pi \times 0.75^2 / 4} = 0.01756 \text{ in } \downarrow$$

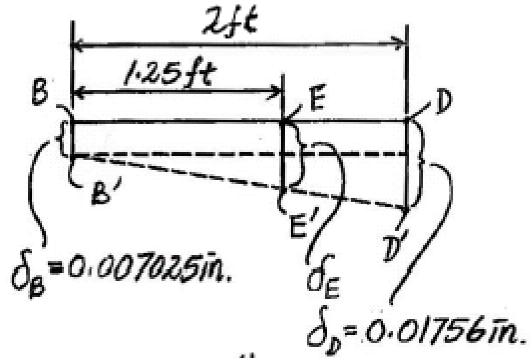


Fig. 2(b)

From the geometry shown in Fig. 2(b):

$$\delta_E = 0.007025 + \frac{1.25}{2}(0.01756 - 0.007025) = 0.01361 \text{ in } \downarrow$$

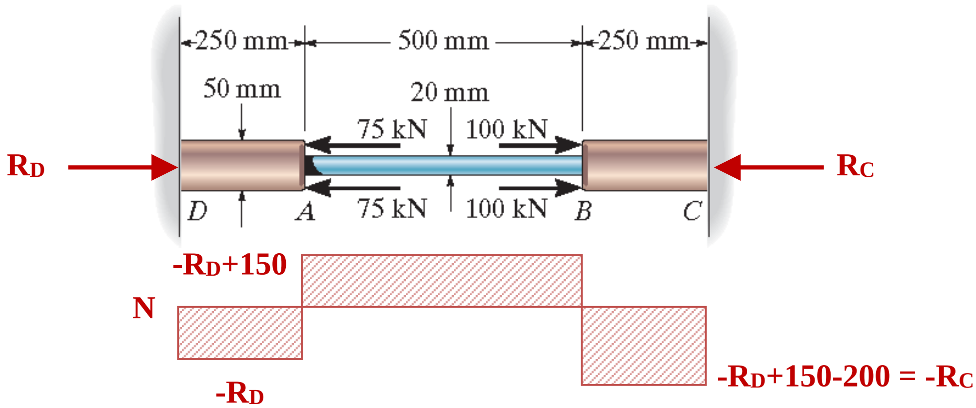
The elongation experienced by bar *EF* is given by:

$$\delta_{F/E} = \frac{F_{EF} L_{EF}}{E_{st} A} = \frac{10 \times 1 \times 12}{29 \times 10^3 \times \pi \times 0.75^2 / 4} = 0.009366 \text{ in } \downarrow$$

Thus, point *F* moves:

$$\delta_F = \delta_E + \delta_{F/E} = 0.01361 + 0.009366 = 0.02298 \text{ in } \downarrow$$

Problem 3. The composite bar in Fig. 3 consists of a 20-mm diameter A-36 steel segment *AB* and 50-mm-diameter red brass C83400 end segments *DA* and *CB*. Determine the displacement of *A* with respect to *B* due to the applied load. E_{st} = 200 GPa, E_{br} = 101 GPa.



Equilibrium: $\sum F_x = R_D - 150 + 200 - R_C = 0 \quad \Rightarrow \quad R_D = R_C - 50$ (1)

Compatibility: $\delta = -\delta_{DA} + \delta_{AB} - \delta_{BC} = 0$ (2)

$$-\frac{N_{DA}L_{DA}}{E_{DA}A_{DA}} + \frac{N_{AB}L_{AB}}{E_{AB}A_{AB}} - \frac{N_{BC}L_{BC}}{E_{BC}A_{BC}} = 0$$

$$-\frac{R_D \times 10^3 \times 250}{101\times 10^3 \times \frac{\pi}{4}50^2} + \frac{(-R_D + 150)\times 10^3 \times 500}{200\times 10^3 \times \frac{\pi}{4}20^2} - \frac{(R_D + 50)\times 10^3 \times 250}{101\times 10^3 \times \frac{\pi}{4}50^2} = 0$$

From (1) and (2): R_D = 107.89 kN, R_C = 157.89 kN

$$\therefore \quad \delta_{AB} = \frac{N_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{42.11\times 10^3 \times 500}{200\times 10^3 \times \frac{\pi}{4}20^2} = \underline{0.335\text{ mm}}$$

<u>Problem 4.</u> The concrete post in Fig. 4 is reinforced using six steel reinforcing rods, each having a diameter of 20 mm. Determine the stress in the concrete and the steel reinforcing bars if the post is subjected to an axial load of 900 kN. E_{st} = 200 GPa and E_c = 25 GPa.

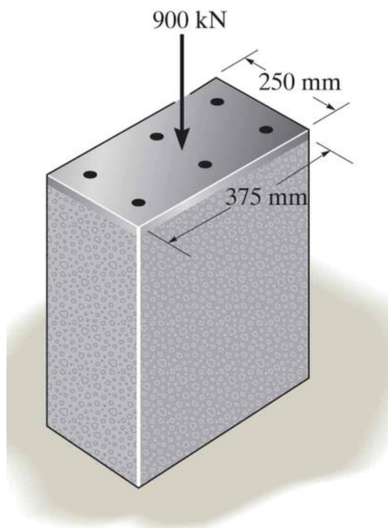


Fig. 4

Establishing equilibrium:

$$\sum F_y = 0 \quad \Rightarrow \quad 900 = F_c + 6F_s \qquad (1)$$

Compatibility of deformation establishes:

$$\delta_c = \delta_s$$

$$\frac{F_c L}{E_c A_c} = \frac{F_s L}{E_s A_s} \qquad (2)$$

The cross-sectional areas of steel and concrete are given by:

$$A_s = \frac{\pi 20^2}{4} = 314.2 \text{ mm}^2$$

$$A_c = (250 \times 375) - (6 \times 314.2) = 91{,}864.8 \text{ mm}^2$$

Substituting (1) into (2) results in:

$$\frac{(900 - 6F_s)}{E_c A_c} = \frac{F_s}{E_s A_s}$$

$$(900 - 6F_s) E_s A_s = F_s E_c A_c$$

$$F_s = \frac{900 E_s A_s}{E_c A_c + 6 E_s A_s} = \frac{900 \times 200 \times 314.2}{(25 \times 91{,}864.8) + (6 \times 200 \times 314.2)} = 21.1 \text{ kN}$$

$$F_c = 900 - 6F_s = 900 - (6 \times 21.1) = 773.4 \text{ kN}$$

The axial stresses in the reinforcing steel bars and concrete are therefore given by:

$$\sigma_s = \frac{F_s}{A_s} = \frac{21.1\times10^3}{314.2} = \underline{67.1\text{ MPa}}$$

$$\sigma_c = \frac{F_c}{A_c} = \frac{773.4\times10^3}{91,864.8} = \underline{8.4\text{ MPa}}$$