STAT 2507 Assignment # 4 (Chapters 9 & 10) Winter 2015
Due in class: Sections E, F and H, April 1 ; Section G, April 2

Last Name ______________________ **First Name** ______________
Student # ______________________ **Lab session:** __________

Total of marks=100.

Part I. Lab questions. Use only the blanks left to answer lab questions.

1. *Confidence interval for μ when σ is known*
   Suppose $n = 9$ people are selected at random from a large population. Assume the heights of the people in this population are normal, with mean $\mu = 68.71$ inches and $\sigma = 3$ inches. Simulate the results of this selection 20 times and in each case find a 90% confidence interval for μ. The following commands may be used:

   MTB > random 9 c1-c20;

   SUBC> normal 68.71 3.

   MTB > zinterval 0.90 3 c1-c20

   a. How many of these intervals do you expect to include $\mu = 68.71$? (20)(0.9)=18

   b. How many of your intervals contain μ? I had 16

   c. Do all the intervals have the same width? Yes. Why (what is the theoretical width)? $2z_{0.05}(3/\sqrt{9}) = 2 * (1.645) = 3.29$.

   d. Suppose you constructed 85% intervals instead of 90%. Would they be narrower or wider? Narrower.

e. How many of your intervals contained the value 71? I had 7

f. Suppose you took samples of size $n = 4$ instead of $n = 9$. Would you expect more or fewer intervals to contain the value 71? More, since smaller sample sizes result in wider confidence intervals. What about 68.71? Same, since 68.71 is the mean. What about the width of the intervals for $n = 4$: Would the width of the intervals with $n = 4$ be narrower or wider than with $n = 9$? Wider.

2. *Confidence interval for μ when σ is NOT known*

Repeat the simulation of Question 1 but now assume σ is unknown and use the t-intervals command to get the 20 90% intervals:

MTB > random 9 c1-c20;

SUBC > normal 68.71 3.

MTB > tinterval 0.90 c1-c20

a. How many of your intervals contain μ? I had 17.

b. Would you expect all 20 of the intervals to contain μ? No. Why? Expected is (20)(0.90)=18.

c. Do all the intervals have the same width? No. Why (what is the theoretical width)? $2(t_{0.05})S/\sqrt{n}$, where the sample standard deviation S changes from sample to sample.

d. Suppose you took 95% intervals instead of 90%. Would they be narrower or wider? Wider.

e. How many of your intervals contain the value 71? I had 6

f. Suppose you took samples of size $n = 64$ instead of $n = 9$. Would you expect more or fewer intervals to contain 71? Fewer. What about 68.71? Same. What about the width of the intervals for $n = 64$: Would they be narrower or wider than for $n = 9$? Narrower.

3. *Hypothesis testing for μ when σ is known*

Imagine choosing $n = 16$ women at random from a large population and measuring their heights. Assume that the heights of the women in this population are normal with $\mu = 63.8$ inches and $\sigma = 3$ inches. Suppose you then test the null hypothesis $H_0: \mu = 63.8$ versus the alternative that $H_a: \mu \neq 63.8$, using $\alpha = 0.10$. Assume σ is known. Simulate the results of doing this test 30 times as follows:

MTB > random 16 c1-c30;

SUBC > normal 63.8 3.

MTB > ztest 63.8 3 c1-c30

a. In how many tests did you reject H_0. That is, how many times did you make an "incorrect decision"? I had 3 p-values less than 0.10.

b. Are the *p*-values all the same for the 30 tests? No.

c. Suppose you used $\alpha = 0.001$ instead of $\alpha = 0.10$. Does this change any of your decisions to reject or not? Yes, some may report No but Yes is more likely. In general, should the number of rejections *increase* or *decrease* if $\alpha = 0.001$ is used instead of $\alpha = 0.10$? Decrease.

d. Now assume that the population really has a mean of $\mu = 63$, instead of 63.8, and carry out the above 30 simulations, (thus, use the above minitab commands with '*normal 63.8 3*' changed to '*normal 63 3*'. Once again, using $\alpha = 0.10$ and assuming σ known, in how many tests did you reject H_0? I had 12 p-values less than 0.10.

A rejection of $H_0: \mu = 63.8$ in part (a) is a "correct decision". True or False? False

A rejection of $H_0: \mu = 63.8$ in part (d) is a "correct decision". True or False? True

4. *Hypothesis testing for μ when σ is NOT known.*

   Repeat Question 3, using *ttest* instead of *ztest*, and answer parts (a), (b), and (c) again. (Thus '*ztest 63.8 3 c1-c30*' changes to '*ttest 63.8 c1-c30*')

   a. In how many tests did you reject H_0. That is, how many times did you make an "incorrect decision"? I had 4.

   b. Are the *p*-values all the same for the 30 tests? No.

   c. Suppose you used $\alpha = 0.00008$ instead of $\alpha = 0.10$. Does this change any of your decisions to reject or not? Yes, some may report No but the answer Yes is more likely. In general, should the number of rejections *increase* or *decrease* if $\alpha = 0.00008$ is used instead of $\alpha = 0.10$? Decrease.

Part II Comprehension questions

1. A fast food franchiser is considering building a restaurant at a certain location. According to a financial analysis, a site is acceptable only if the number of pedestrians passing the location

averages more than 100 per hour. A random sample of 50 hours produced $\bar{x} = 110$ and $s = 12$ pedestrians per hour.

(a) Do these data provide sufficient evidence to establish that the site is acceptable? Use $\alpha = 0.05$.

(b) What are the consequences of Type I and Type II errors? Which error is more expensive to make?

(c) Considering your answer in part (b), should you select α to be large or small? Explain.

(d) What assumptions about the number of pedestrians passing the location in an hour are necessary for your hypothesis test to be valid?

Solution: **a.** $H_0 : \mu = 0$ vs. $H_a : \mu > 100$, since n is large enough, test statistic is $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{110 - 100}{12/\sqrt{50}} = 5.89$, null hypothesis is rejected since $5.89 > 1.645 = z_{0.05}$

b. Type I error is rejecting H_0 when it is true and the result of this error is constructing the site when there are not enough pedestrians passing by the location (bankruptcy). Type II error is to accept H_0 when it is not true, so in this case the site is not constructed in location with more than 100 pedestrians passing by (missing a good location). Type I error is more expensive. (Some may say that type II error is more expensive. Type II error is also accepted as a correct answer if it supported by a good explanation)

c. α should be a small number.

d. None.

2. An experiment was conducted to test the effect of a new drug on a viral infection. The infection was induced in 100 mice, and the mice were randomly split into two groups of 50. The first group, the *control group*, received no treatment for the infection. The second group received the drug. After a 30-day period, the proportions of survivors, $\hat{p}_1$ and $\hat{p}_2$, in the two groups were found to be 0.36 and 0.60, respectively.

   (a) Is there sufficient evidence to indicate that the drug is effective in treating the viral infection? Test at 5% significance level. (Make sure to state your <u>null and alternative</u> hypotheses.)
   Solution: Solution: Let p_1 = Proportion of Survivors in control group
   p_2 = Proportion of Survivors in treatment group. In this part we wish to test

$$\begin{cases} H_0 : p_1 - p_2 = 0, & \text{(Not Effective)} \\ H_a : p_1 - p_2 < 0, & \text{(Effective)} \end{cases}$$

Observe the $n_1\hat{p}_1, n_1(1-\hat{p}_1), n_2\hat{p}_2, n_2(1-\hat{p}_2) > 5$. Also observe that $\hat{p}_1 = 0.36$, hence $\frac{x_1}{50} = \frac{36}{100}$, i.e., $x_1 = 18$ and
$\hat{p}_2 = 0.6$, hence $\frac{x_2}{50} = \frac{60}{100}$, i.e., $x_2 = 30$. Therefore $\hat{p} = \frac{x_1+x_2}{n_1+n_2} = \frac{18+30}{100} = 0.48$ and as a result $\hat{q} = 1 - \hat{p} = 0.52$. So, the test statistic is of the form

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.36 - 0.6}{\sqrt{(0.48)(0.52)\frac{2}{50}}} = -2.4$$

. The critical value is $-z_{0.05} = -1.645$. Since $-2.4 < -1.645$, we reject H_0.

(b) Use a 95% confidence interval to estimate the actual difference in the cure rates, i.e. $p_1 - p_2$, for the treatment versus the control groups. Based on this confidence interval can you conclude that the drug is effective? Why?

Solution: It is important to note that, roughly speaking, here the 95% confidence interval for $p_1 - p_2$ is in fact the acceptance region of the test $H_0 : p_1 - p_2 = 0$ versus $H_a : p_1 - p_2 \neq 0$ at level $\alpha = 0.05$. And the confidence interval is

$$\hat{p}_1 - \hat{p}_2 \pm 1.96\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = 0.36 - 0.6 \pm \sqrt{\frac{(0.36)(0.64)}{50} + \frac{(0.6)(0.4)}{50}} = \Big[-0.4301, -0.0498\Big].$$

Since 0 is not in the confidence interval (i.e., the value of the the test statistic is in the rejection region), we reject H_0 at level $\alpha = 0.05$.

3. In an investigation of pregnancy-induced hypertension, one group of women with this disorder was treated with low-dose aspirin, and a second group was given a placebo. A sample consisting of 23 women who received aspirin has mean arterial blood pressure 111 mm Hg and standard deviation 8 mm Hg; a sample of 24 women who were given the placebo has mean blood pressure 109 mm Hg and standard deviation 8 mm Hg.

(a) At the 0.01 level of significance, test the null hypothesis that the two populations of women have the same mean arterial blood pressure. Justify any procedure you use.
Solution: Let $\mu_1 =$ the average blood pressure of the aspirin group
$\mu_2 =$ the average blood pressure of the placebo group
Observe that $\frac{S_1^2}{S_2^2} \leq 3$ and that $S_P^2 = \frac{22\times64+23\times64}{23+24-2} = 64$. We wish to test $H_0 : \mu_1 - \mu_2 = 0$

versus $H_a : \mu_1 - \mu_2 \neq 0$. The test statistic is

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S_P^2(\frac{1}{23} + \frac{1}{24})}} = 0.86$$

The critical value $t_{45,0.005} = 2.576$. Since $0.86 < 2.576 \rightarrow$ We do not reject H_0.

(b) Construct a 99% confidence interval for the true difference in population means. Does this interval contain the value 0? Based on this confidence interval, what is you conclusion regarding the effect of the two treatments on the blood pressure of pregnant women?
Solution: The 95% confidence interval for $\mu_1 - \mu_2$ is

$$\left[\bar{x}_1 - \bar{x}_2 - t_{45,0.005}\sqrt{S_P^2(\frac{1}{23} + \frac{1}{24})}, \bar{x}_1 - \bar{x}_2 + t_{45,0.005}\sqrt{S_P^2(\frac{1}{23} + \frac{1}{24})}\right] = \left[-0.995, 4.995\right]$$

Now since $0 \in \left[-0.995, 4.995\right]$, we do not reject H_0.

4. In an attempt to compare the starting salaries for university graduates who majored in education and the social sciences, random samples of 100 recent university graduates were selected from each major and the following sample information was obtained:

Major	Mean	St. Dev.
Education	\$50,554	\$2225
Social Science	\$48,348	\$2375

Conduct an appropriate hypothesis test at the 5% level of significance to determine if there is a difference in the average starting salaries for all university graduates who majored in education and the social sciences. Conduct this test using

(a) the p-value method,

(b) the critical value method , and

(c) the confidence interval method .

Solution: Let μ_1 = the average starting sealery of Education graduates
μ_2 = the average starting sealery of Education graduates. We wish to test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_a : \mu_1 - \mu_2 \neq 0$.

a. Observe that in this problem the sample sizes $n_1, n_2 \geq 30$. Hence the test statistics is

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{50554 - 48348}{\sqrt{\frac{(2225)^2}{100} + \frac{(2375)^2}{100}}} = 6.8$$

$p - value = P(Z > 6.8) + P(Z < -6.8) \approx 0 + 0 = 0$ (we use the normal table here!). Since $p - value < 0.05 = \alpha$, we reject H_0.

b. The critical values here are $z_{.025} = 1.96$ and $-z_{.025} = -1.96$ (from the normal table). Since the value of the test statistic is 6.8 and $6.8 > 1.96$, we reject H_0.

c. The 95% confidence interval is

$$\bar{x}_1 - \bar{x}_2 \pm 1.96\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = 2206 \pm 1.96 \times 324.44 = \left[1568.137, 2843.862\right].$$

Since, 0 is not in the confidence interval $\left[1568.137, 2843.862\right]$, we reject H_0 in favor of H_a at level 5%.

5. A company is interested in offering its employees one of two employee benefit packages. A random sample of the company's employees is collected, and each person in the sample is asked to rate each of the two packages on an overall preference scale of 0 to 100. Results were

Employee	Program A	Program B
1	45	56
2	67	70
3	63	60
4	59	45
5	77	85
6	69	79
7	45	50
8	39	46
9	52	50
10	58	60
11	70	82

At significant level $\alpha = 0.05$, do you believe that the employees of this company prefer, on the average, one package over the other? State the Null and Alternative hypotheses and show the calculations that you use to draw a conclusion.

Solution:

We wish to test $H_0 : \mu_1 - \mu_2 = 0$ vs. $H_a : \mu_1 - \mu_2 \neq 0$ or $H_0 : \mu_d = 0$ vs. $H_a : \mu_d \neq 0$

It is a paired test where d_i: $-11, -3, 3, 14, -8, -10, -5, -7, 2, -2, -12$

$\bar{d} = \sum d_i/n = -39/11 = -3.55$ and $s_d^2 : \frac{\sum d_i^2 - (\sum d_i)^2/n}{n-1} = \frac{725-138.27}{10} = 58.67$

The test statistic is $t = \dfrac{\bar{d}-0}{s_d/\sqrt{n}} = \dfrac{-3.55-0}{7.66/\sqrt{11}} = -1.54$ and $-t_{0.025;10} = -2.228$, so H_0 is not rejected. In other words, at level $\alpha = 0.05$, there is no significant evidence to believe that the employees prefer one package over the other one.