STAT 2507 H

Assignment # 2 Solutions

(Chapters 4, 5, and 6)

Due: Monday, March 2, 2015, in class

Last Name ______________________ **First Name** ______________________

Student # ______________________ **Your LAB Section** ______________________

Note: Use spaces left to answer all questions. The total of marks for the questions is 100.

Part I: Lab Questions

1. **[8]** Suppose that X has a binomial distribution with n = 31 and p = 0.7. Using the following Minitab command, answer (a) - (c).
   cdf;
   binomial 31 0.7.
   (a)[3] $P(X<20) = \underline{P(X \leq 19) = 0.1924}$ and $P(X \leq 20) = \underline{0.3121}$

   (b)[3] $P(X>20) = \underline{1 - P(X \leq 20) = 1 - 0.3121 = 0.6879}$
   and $P(X \geq 20) = \underline{1 - P(X \leq 19) = 1 - 0.1924 = 0.8076}$

   (c)[2] $P(18<X<23) = \underline{P(X \leq 22) - P(X \leq 18) = 0.6135 - 0.1069 = 0.5066}$

2. **[9]**Suppose that X be a Poisson random variable with mean $\mu = 3$. Answer (a) - (c) using following Minitab command.
   cdf;
   Poisson 3.
   (a)[3] $P(X = 2) = \underline{P(X \leq 2) - P(X \leq 1) = 0.4232 - 0.1992 = 0.2240}$

   (b) [3] $P(2<X \leq 4) = \underline{P(X \leq 4) - P(X \leq 2) = 0.8153 - 0.4232 = 0.3921}$

   (c) [3] $P(2 \leq X<5) = \underline{P(X \leq 4) - P(X \leq 1) = 0.8153 - 0.1992 = 0.6161}$

3. **[6]**Suppose that X has a hypergeometric distribution with parameters N = 32, M = 12 and n = 7.
   (a)[3] Using the following Minitab command, calculate the below probabilities.
   cdf 4;
   hypergeometric 32 12 7.
   $P(X \leq 4) = \underline{0.9496}$ and $P(X>4) = \underline{1 - P(X \leq 4) = 1 - 0.9496 = 0.0504}$

   (b) [3] Using the following Minitab command, calculate the value of "a" such that $P(X \leq a)>0.8$ for above hypergeometric distribution.
   invcdf 0.8;
   hypergeometric 32 12 7.
   $a = \underline{4}$

4. [7]Suppose that X has a normal distribution with mean, $\mu = 12$ and variance, $\sigma^2 = 9$.
(a) **[5]** Calculate the given probabilities using below Minitab command.
cdf x;
normal 12 3.
[2]$P(X>14) = 1 - P(X \leq 14) = 1 - 0.7475 = 0.2525$
and **[3]** $P(13 \leq X \leq 18) = P(X \leq 18) - P(X \leq 13) = 0.9773 - 0.6306 = 0.3467$

(b) **[2]** Calculate the value of a such that $P(X \leq a) = 0.44$, using the following Minitab command when $X \sim N(12, 9)$.
invcdf 0.44;
normal 12 3.

$P(X \leq 11.55) = 0.44$.

Part II: Long Answer Questions

1. **[10]** Calculate the corresponding probabilities
(a) **[5]** Suppose that $P(A) = 0.6$, $P(B) = 0.7$, and that events A and B are independent. Find [2]$P(A \bigcap B)$, [1]$P(A \bigcup B)$, [1]$P(A|B)$ and **[1]**$P(B|A)$.
(b) **[5]** Suppose that P(A) = 0.4, P(B) = 0.5, and that events A and B are mutually exclusive. Find [1]$P(A \bigcap B)$, [2]$P(A \bigcup B)$, [1]$P(A|B)$ and **[1]**$P(B|A)$.

Solution:
(a) $P(A \bigcap B) = P(A) * P(B) = 0.6 * 0.7 = 0.42$ as A and B are independent;
$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B) = 0.6 + 0.7 - 0.42 = 0.88$
$P(A|B) = P(A) = 0.6$ as A and B are independent
$P(B|A) = P(B) = 0.7$ as A and B are independent.
(b) $P(A \bigcap B) = 0$ as A and B are mutually exclusive;
$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B) = 0.4 + 0.5 - 0 = 0.9$;
$P(A|B) = \frac{P(A \bigcap B)}{P(B)} = 0$;
$P(B|A) = \frac{P(A \bigcap B)}{P(A)} = 0$.

2. **[10]** The random variable x is defined as the number of mistakes made by a typist on a randomly chosen page of a physics thesis. The probability distribution follows:

x	0	1	2	3
$p(x)$	0.3	0.40	0.20	?

(a) **[4]** Calculate **[1]**$p(3)$, **[1.5]**the expected of number of mistakes per page and the corresponding **[1.5]**standard deviation.
(b) **[2]** Find the following probabilities: $P(X<1)$ and $P(0 \leq X<2)$.
(c) **[4]** In what fraction of pages in the thesis would the number of mistakes made be within

two standard deviations of the mean?

Solution:
(a) $p(0) + p(1) + p(2) + p(3) = 1 \Rightarrow p(3) = 1 - p(0) - p(1) - p(2) = 1 - 0.3 - 0.4 - 0.2 = 0.10$
$E(X) = \mu = \sum xp(x) = (0)(0.3) + (1)(0.4) + (2)(0.2) + (3)(0.1) = 1.1$
$Var(X) = \sigma^2 = \sum(x-\mu)^2 p(x) = (0-1.1)^2(0.3) + (1-1.1)^2(0.4) + (2-1.1)^2(0.2) + (3-1.1)^2(0.1) = 0.890 \Rightarrow \sigma = 0.9434$
(b) $P(X<1) = P(X=0) = 0.3$
$P(0 \le X<2) = P(X=0) + P(X=1) = 0.3 + 0.4 = 0.7$
(c) $\mu \pm 2\sigma = 1.1 \pm (2)(0.9434) = 1.1 \pm 1.8868 \Rightarrow$ -0.7868 to 2.9868. Values taken by random variable X in this interval are 0, 1, 2. Hence, the fraction of pages where $x = 0, 1, 2$ is $p(0) + p(1) + p(2) = 0.9$

3. **[10]** Steve takes either a bus or the subway to go to work, with probabilities 0.25 and 0.75, respectively. When he takes the bus, he is late 40% of the time. When he takes the subway, he is late 30% of the time. If Steve is late for work on a particular day, what is the probability that he took the bus?

Solution:
Define the following events:
B: Steve takes the bus.
S: Steve takes the subway.
L: Steve is late for work.
It is given that $P(B) = 0.25$, $P(S) = 0.75$, $P(L/B) = 0.40$, and $P(L/S) = 0.30$.
Using Bayes' Rule,
$P(B|L) = \frac{P(L|B)P(B)}{P(L|B)P(B)+P(L|S)P(S)} = \frac{(0.40)(0.25)}{(0.40)(0.25)+(0.30)(0.75)} = 0.3077$

4. **[10]** Of all the Harry Potter books purchased in a recent year, about 60% were purchased for readers 14 or older. If 12 Harry Potter fans who bought books that year are surveyed, find the following probabilities.
(a) **[3]** At least five of them are 14 or older.
(b) **[4]** Exactly nine of them are 14 or older.
(c) **[3]**Less than three of them are 14 or older.

Solution: Let X be the number of readers who are 14 or older. Distribution of X is binomial with number trials, $n = 12$ and $p = 0.6$
(a) $P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.057 = 0.943$
(b) $P(X = 9) = P(X \le 9) - P(X \le 8) = 0.917 - 0.775 = 0.142$
(c) $P(X<3) = P(X \le 2) = 0.003$

5. **[10]** In a food processing and packaging plant, there are, on average, 2 packaging machine breakdowns per week.

(a) **[3]** Calculate the probability that there are no more than two machine breakdowns in a given week.

(b) **[3]** What is the probability that there are no machine breakdowns in a given week?

(c) **[4]** In a random sample 26 machines, what is the probability that there are no machine break downs in at least one of them in a given week?

Solution:

(a) $P(X \leq 2) = \frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} = 0.1353 + 0.2707 + 0.2707 = 0.6767$

(b) $P(X = 0) = \frac{2^0 e^{-2}}{0!} = 0.1353 \approx 0.14$

(c) Let Y be the number of machines without breakdowns. Distribution of Y is binomial with number of trials, $n = 26$ and $p = P(X = 0) = 0.14$

$P(Y \geq 1) = 1 - P(Y = 0) = 1 - C_0^{26}(0.14)^0(1 - 0.14)^{(26-0)} = 1 - 0.0198 = 0.9802$

6. **[10]** Heidi prepares for an exam by studying a list of 15 problems. She can solve 9 of them. For the exam, the instructor selects 7 questions at random from the list of 15. What is the probability that Heidi can solve all 7 problems on the exam?

Solution:

Let X be the number problems that can be solved by Heidi. X has hypergeometric distribution with N = 15, M = 9, n = 7.

$P(X = 7) = \frac{C_7^9 C_0^6}{C_7^{15}} = 36/6435 = 0.0056$? $\frac{9!}{7!\,2!}$ $\binom{6}{0} = 1.$ $\frac{15!}{7!\,8!}$

7. **[10]** Suppose the amount of heating oil used annually by households in Ontario is normally distributed, with a mean of 760 litres per household per year and a standard deviation of 150 litres of heating oil per household per year.

(a) **[3]** What is the probability that a randomly selected Ontario household uses more than 570 litres of heating oil per year?

(b) **[3]** What is the probability that a randomly selected Ontario household uses between 680 and 1130 litres per year?

(c) **[4]** If the members of a particular household were scared into using fuel conservation measures by newspaper accounts of the probable price of heating oil next year, and they decided they wanted to use less oil than 97.5% of all other Ontario households currently using heating oil, what is the maximum amount of oil they can use and still accomplish their conservation objective?

Solution:

Let X be the litres of heating oil per household per year. Given that $X \sim N(760, 150^2)$. **(a)** $P(X{>}570) = P(\frac{X-\mu}{\sigma} > \frac{570-760}{150}) = P(Z > -1.27) = 0.8980$ (0.1020)

(b) $P(680 \leq X \leq 1130) = P(\frac{680-760}{150} \leq \frac{X-\mu}{\sigma} \leq \frac{1130-760}{150}) = P(-0.53 \leq Z \leq 2.47) = P(Z \leq$

$2.47) - P(Z \leq -0.53) = 0.9932 - 0.2981 = 0.6951$

(c) Let x_0 be maximum amount of oil they can use and still accomplish their conservation objective.

$P(X > x_0) = 0.975 \Rightarrow P(Z > \frac{x_0 - 760}{150}) = 0.975 \Rightarrow \frac{x_0 - 760}{150} = -1.96 \Rightarrow x_0 = 466$ litres of heating oil per year.