Concordia University

Course	Number	Sections
ENGR	233	P, Q

Examination	Date	Time	Total Marks	Pages
Final	April 2008	3 hours	100	2

Course Coordinator	Instructors
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Special Instructions: use of calculators and outside materials is NOT permitted.

Each problem is worth 10 marks unless stated otherwise.

Problem 1. For the vector field

$$\vec{F}(x,y,z) = x^2y\mathbf{i} + xy^2\mathbf{j} + 2xyz\mathbf{k}$$

compute –**if possible**– the following quantities. If it is not possible **explain why not**.

(a) div(curl $\vec{F}(x,y,z)$) , **(b)** curl(div $\vec{F}(x,y,z)$) , **(c)** grad(div $\vec{F}(x,y,z)$) , **(d)** div(grad $\vec{F}(x,y,z)$)

Problem 2. Find the equation of the tangent plane of the surface defined by

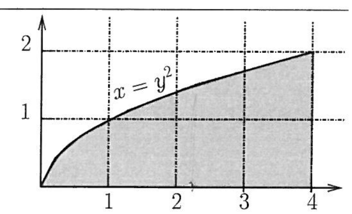
$$z^3 - xyz = 1$$

at the point $(4, \frac{1}{2}, -1)$.

Problem 3.
Evaluate the following integral by reversing the order of integration

$$\int_0^2 \int_{y^2}^4 e^{\sqrt{x^3}} dx dy$$

[Hint: the following substitution may be of help: $u = x^{\frac{3}{2}}$]



Problem 4. Find the rate of change at the point $(2, 1, 3)$ of the following function $f(x,y,z) = \frac{xy}{z^2}$ along the directions given by unit vectors parallel to
(a) $\mathbf{i}$; **(b)** $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

Problem 5. Using **Stokes' theorem**, compute the flux of the curl of the vector field

$$\vec{F}(x,y,z) = 6yz\mathbf{i} - 24x\mathbf{j} + yze^{x^2+\arctan(z)}\mathbf{k}$$

across the surface S (oriented upwards) of the paraboloid $z = y^2 + x^2$, $z \leq 4$, with boundary the circle $z = 4, x^2 + y^2 = 4$.

Problem 6. Find the mass $M = \iiint_{\mathcal{R}} \rho(x,y,z)dV$ of the solid in the first octant (namely $x \geq 0, y \geq 0, z \geq 0$) bounded by the coordinate planes and the graph of $x + y + z = 1$ if the density is given by $\rho = x + 2y$.

Problem 7.
Evaluate the work done by the **conservative** force

$$\vec{F}(x,y) = ye^{xy}\mathbf{i} + (xe^{xy} + 2y)\mathbf{j}$$

along any path that joins the starting point $(0,0)$ and ending point $(1,2)$. **You must use the potential function.**

Problem 8. Find the curvature $\varkappa(t)$ and the components of the acceleration $a_N(t)$; $a_T(t)$ for the curve described by

$$\mathbf{r}(t) = (t+1)\mathbf{i} + (t^2 - t)\mathbf{j} + \mathrm{e}^{-t}\mathbf{k}$$

Problem 9. Use **Green's theorem** to compute the line-integral

$$\oint_C y^2 \mathrm{d}x + x\mathrm{d}y$$

where C is the boundary of the region determined by the graphs of $x = 0$, $x^2 + y^2 = 4$ and with $x \geq 0$.

Problem 10. Use the **Divergence Theorem** to evaluate the outward flux $\iint_S \vec{F} \cdot \vec{n}\mathrm{d}S$ of the given vector field across the surface specified

$$\vec{F}(x,y,z) = x^3\mathbf{i} + (y^3 + xz)\mathbf{j} + (z^3 + z^2)\mathbf{k}$$

$$x^2 + y^2 + z^2 = a^2, \quad a > 0.$$
