Concordia University

Course				Number
ENGR				233

Examination	Date	Time	Total Marks	Pages
Final	April 2007	3 hours	100	2

Course Coordinator	Instructors
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Special Instructions: use of calculators and outside materials is NOT permitted.

Each problem is worth 10 marks unless stated otherwise.
A one page formula sheet will be handed in during the final exam.

Problem 1. Let us denote by $h(x, y, z)$ a scalar function and $\vec{F}(x, y, z)$ a vector field. Identify which of the following operations are not allowed and which ones make mathematical sense. Write **(Yes)** when the expression is a valid one or **(No)** next to each letter in your booklet. In case the expression is not valid.
Credit will NOT be given for correct answers unless reasons or justifications (one sentence) are shown in the examination booklet.
(**example**: "The expression is not defined because the ..... of a scalar/vector function is not defined." or "The expression is well defined because the .... of a scalar is defined.").

(**a**) grad (curl h) (**b**) grad (curl $\vec{F}$) (**c**) div (curl $\vec{F}$) (**d**) grad (div $\vec{F}$) (**e**) grad (div h)
(**f**) div (curl (grad h)) (**g**) curl (div (grad h)) (**h**) div (curl (grad $\vec{F}$))
(**i**) curl (div (grad $\vec{F}$)) (**j**) grad (div (grad h))

Problem 2. Find the equation of the tangent plane to the graph of the equation

$$z = 2 - x^3 + y^2$$

at the point $(3, -4, -9)$.

Problem 3. Set up, but **do not evaluate** the double integral of $g(x, y) = \mathrm{e}^{-x^2+y}$ over the region bounded by the curves

$$y = \sqrt{1 - x^2}\ , \quad y = \sqrt{4 - x^2}\ , \quad y = x\ , \quad x = 0$$

using **polar coordinates**. In particular you must write the integral as a suitable iterated integral in the r, θ coordinates.

Problem 4. Find the directions in which the following function has the maximum and minimun rates of change at the point $(1, 0, 1)$. **Find those rates**.

$$F(x, y, z) = x^3 + y^2 x + z^2 x + y x^2 + y^3 + z^2 y$$

Problem 5. Using the **divergence theorem**, compute the flux of the vector field

$$\vec{F}(x, y, z) = (2x - y^2 \cos(z))\mathbf{i} + (y - \ln(1 + x^2))\mathbf{j} + \mathrm{e}^{x^2 y^3}\mathbf{k}$$

across the surface of the sphere S of radius 5 centered at $(0,3,1)$ (HINT: the volume of the sphere of radius R is $\frac{4\pi}{3}R^3$)

Problem 6. Using Green's theorem, compute the following line integral

$$\oint_C (-2y + e^{x^2})dx + \left(4x^2 - \ln(1+y^2)\right) dy$$

where C is the boundary of the rectangle of vertices $(0,0),(1,0),(1,3),(0,3)$ traversed counterclockwise.

Problem 7.
Evaluate the work done by the **conservative force**

$$\mathbf{F} = (2xy + ze^x)\,\mathbf{i} + x^2\mathbf{j} + e^x\mathbf{k}$$

along the path

$$\mathbf{r}(t) = 2t\mathbf{i} + (1+\cos(t))^2\mathbf{j} + 4\sin^3(t)\mathbf{k}\ , \qquad 0 \le t \le \frac{\pi}{2}$$

Problem 8. Find the curvature of the elliptical helix described by

$$\mathbf{r}(t) = a\cos(t)\mathbf{i} + b\sin(t)\mathbf{j} + ct\mathbf{k}$$

where $a > 0, b > 0, c > 0$ are arbitrary constants.

Problem 9. In Maxwell's equations of electrodynamics the density of charge ρ and the electric vector field $\mathbf{E}$ are related by the equation

$$\text{div } \mathbf{E} = \rho$$

Knowing that in a region of space $\mathbf{E} = x\mathbf{i} + y^3\mathbf{j} + z^2\mathbf{k}$, find
(a) the density of charge $\rho(x,y,z)$ and
(b) find the total charge $Q = \iiint_{\mathcal{R}} \rho\, dV$ in the box

$$\mathcal{R} = \{0 \le x \le 2\ ,\quad 0 \le y \le 1\ ,\quad -1 \le z \le 1\}$$

Problem 10. Rewrite the following integral in **cylindrical coordinates, and evaluate it**

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{1-y^2}} \int_0^{1+x^2+y^2} \frac{1}{\sqrt{x^2+y^2}} dz dx dy$$
