Assignment Four - Part two
Sampling Distributions

Part I of the assignment :

- *Must be done via MySytatlab*
- *Due-date: Monday, July 27, 11:59 P.M.*

Part II of the assignment :

- *Please don't forget to complete your signed statement of Academic Integrity within the body of your solution*
- *Submit a PDF of your type-written (i.e., not handwritten) solution (recall that a submission cannot be marked unless it is in PDF format).*
- *Submit your assignment via blackboard learn by due-date Monday, July 27, 11:59 P.M.*
- *Make sure to provide detailed calculations and steps of how you arrived at your answer whenever needed.*

Question 1.

When a truckload of apples arrives at a packing plant, a random sample of 150 is selected and examined for bruises, discoloration, and other defects. The whole truckload will be rejected if more than 5% of the sample is unsatisfactory. Suppose that 8% of the apples on the truck do not meet the desired standard. What is the probability that the shipment will be accepted anyway ?

Check the condition first :

Success/Failure Condition: *$np = 12$ and $nq = 138$ are both greater than 10. Therefore, the sampling distribution model for $\hat{p}$ is Normal, with:*

$\mu_{\hat{p}} = 0.08$

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.08)(0.92)}{150}} \cong 0.022$

According to the Normal model, the probability that less than 5% of the apples in the sample are unsatisfactory is approximately 0.088.

$$P(\hat{p} < 0.05) = \frac{0.05 - 0.08}{\sqrt{\frac{(0.08)(0.92)}{150}}} \cong 0.088$$

Question 2.

The average January temperature in Calgary varies from year to year. These average January temperatures themselves have an average, over many past years, of -7.9 degrees Celsius, with a standard deviation of 5.0 degrees. Assume a Normal model applies.

a) During what percentage of years does Calgary have a January average above 0 degrees ?

Let X denote the average January temperature in Calgary in any year. We have $X \sim N(7.9, 5)$.

$$P(X > 0) = P(Z > \frac{0-(-7.9)}{5} = 1 - 0.942 = 0.0571, or\ 5.71\%.$$

b) In the 20% coldest Januaries, the average January temperature reaches no higher than how many degrees ?

We want the value of x such that $P(X \leq x) = 0.20$. The standardized value of x is 0.84. That is :

$$\frac{x-(-7.9)}{5} = 0.84\ or\ x = 7.9 + 5(0.84) = 12.1$$

In the 20% coldest Januaries, the average January temperature gets no higher than 12.1 degrees.

c) Suppose you will be in Calgary for the next four years. Describe the sampling distribution model for the average of the next four January average temperatures. Assume climatic conditions are not changing.

If we assume that the average January temperatures in different years are independent and have the same Normal distribution, (i.e., Normal with mean 7.9 and standard deviation 5), then the average of the next four January average temperatures will have a normal distribution with mean 7.9 and standard deviation $5/\sqrt{4} = 2.5$ degrees Celsius.

d) What is the probability that the average of the next four January average temperatures will exceed -4 degrees ? If this happens, would you consider this evidence of climactic warming (at least locally in Calgary), or just Normal variations ?

$$P(\bar{X} > -4) = P(Z > \frac{-4-(-7.9)}{2.5}) = 10.9406 = 0.0594.$$

This probability is somewhat small but not very surprisingly small (a bit over 5%). It is possible (a chance of approximately 1 in 20) for the average to be higher than 4 without any change in the distribution, i.e., with no climatic warming.

Question 3.

STAT2303 is a large introductory statistics course at the University of Toronto, with 720 students enrolled. On the term test, the average score was 72, with a standard deviation of 16. There are 24 tutorials, with 30 students in each. David teaches one and Jonathan teaches another. Assume that tutorial groups are formed quite randomly.

a) We select a student at random from Jonathan's tutorial. Find the probability that this student's test score is at least 80. Assume that scores are Normally distributed.

Let X denote the test score of this student. Then $X \sim N(\mu = 72, \sigma = 16)$.

$z = (8072)/16 = 0.5$ *and* $P(X > 80) = 10.6915 = 0.3085$.

b) We select one student at random from each tutorial. Find the probability that Jonathan's student outperformed David's student by at least 7 marks on the test. Assume that scores are Normally distributed. (Hint: work with the difference between two scores)

Let Y denote the test score of the student from Davids tutorial. We want :

$$P(X - Y \geq 7)$$
$$X - Y \sim N(72 - 72, \sqrt{16^2 + 16^2} = 22.63)$$
$$P(X - Y \geq 7) = P(Z \geq \frac{7 - 0}{22.63}) = P(Z \geq 0.31) = 1 - 0.6217 = 0.3783.$$

c) What is the probability that the average for Jonathan's tutorial is at least 80 ? Do you need to assume that scores are Normally distributed?

Let $\bar{X}$ denote the average test score in Jonathan's tutorial.

$$\bar{X} \sim N(72, \frac{16}{\sqrt{30}} = 9.29)$$
$$P(X \geq 80) = P(Z \geq \frac{80 - 72}{2.92}) = 1 - 0.9969 = 0.0031$$

d) What is the probability that the average test score in Jonathan's tutorial is at least seven marks higher than the average test score for David's tutorial? Do you need to assume that scores are Normally distributed ? (Hint: Work with the difference between the two tutorial averages.)

Let $\bar{Y}$ denote the average test score in David's tutorial. $\bar{Y}$ also has the same distribution as $\bar{X}$.

$$\bar{X} \sim N(72, \frac{16}{\sqrt{30}} = 9.29)$$
$$\bar{X} - \bar{Y} \sim N(72 - 72, \sqrt{\frac{16^2}{30} + \frac{16^2}{30}})$$
$$P(\bar{X} - \bar{Y} \geq 7) = P(Z \geq \frac{7 - 0}{4.13}) = 1 - 0.9545 = 0.0455.$$