Assignment two - Solution

Probability Theory

Part I of the assignment :

- *Must be done via MySytatlab*
- *Due-date: Monday, June 15, 11:59 P.M.*

Part II of the assignment :

- *Please don't forget to complete your signed statement of Academic Integrity within the body of your solution*
- *Submit a PDF of your type-written (i.e., not handwritten) solution (recall that a submission cannot be marked unless it is in PDF format).*
- *Submit your assignment via blackboard learn by due-date Monday, June 15, 11:59 P.M.*
- *For this assignment you don't require to use MiniTab, Excel or any statistical tool*
- *Make sure to provide detailed calculations and steps of how you arrived at your answer whenever needed.*

Question 1.

Canada conducts a census every five years. Federal, provincial, and local governments use census data to shape public policy. In 2006, Canada tried something no other country had ever tried - census questionnaires that could be completed online. Sixty-six percent of households completed their census questionnaire by the May 16 deadline. (Households that did not respond by the deadline received follow-up letters and phone calls.)

Nineteen percent of households responded online ; sixteen percent of households responded on time and online

Denote the two events as T (on time) and L (online) respectively.
We know that $P(T) = 0.66$, $P(L) = 0.19$, $P(T \text{ and } L) = 0.16$.

a) What percent of households filed on time or filed online ?

$$P(T \text{ or } L) = P(T) + P(L) - P(T \text{ and } L) = 0.66 + 0.19 - 0.16 = 0.69(69\%)$$

b) What percent of households did not file on time but used the online service when they eventually responded ?

$$P(L) - P(L \text{ and } T) = 0.19 - 0.16 = 0.03(3\%)$$

c) What percent of households filed on time but did not file online ?

$$P(T) - P(L \text{ and } T) = 0.66 - 0.16 = 0.50(50\%)$$

d) What percent of households neither filed on time nor online ?

$$P(T^c \text{ and } L^c) = P(T \text{ or } L)^c = 1 - P(T \text{ or } L) = 1 - 0.69 = 0.31(31\%)$$

Question 2.

Part one - Seventy-six percent of Ottawans speak English only when they're at home, 10% speak only French, 11% speak a non-official language, and the remainder speak more than one language. You are conducting a poll by calling Ottawans at random. In your first three calls, what is the probability you talk to :

a) Only people who speak only English at home ?

$0.76 \times 076 \times 0.76 = 0.438976$

b) No people who speak only French at home ?

$0.9 \times 0.9 \times 0.9 = 0.729$

c) At least one person who speaks a non-official language at home ?

P(at least one person speaks a non-official language at home) =
1 - P(no one speaks a non-official language) $= 1 - (1 - 0.11) \times (1 - 0.11) \times (1 - 0.11) = 0.295031$

Part two - Sixty percent of Ottawans drive their own vehicles to work, 7% ride as passengers in someone else's vehicle, 22% take public transit, 10% walk or cycle, and the remainder get to work some other way. Again you are conducting poll by calling Ottawans at random. In your first four calls, what is the probability you talk to :

a) Only people who drive their own vehicles to work ?

$0.6 \times 0.6 \times 0.6 \times 0.6 = 0.1296$

b) No one who takes public transit to work ?

$(1 - 0.22) \times (1 - 0.22) \times (1 - 0.22) \times (1 - 0.22) = 0.37015056$

c) At least one person who walks or cycles to work ?

P(at least one person walks or cycles) =
1 - P(none walks or cycles) $= 1 - (1 - 0.1)(1 - 0.1)(1 - 0.1)(1 - 0.1) = 0.3439$

Question 3.

A survey of students in a large Statistics for Management class asked about their birth order (1 = oldest or only child) and which faculty of the university they were studying under. Here are the data :

	Cost per ounce		
Faculty	**1 or only**	**2 or more**	**Total**
Art & Sciences	34	23	57
Agriculture	52	41	93
Human Ecology	15	28	43
Other	12	18	30
Total	**113**	**110**	**223**

a) If we select a student at random, what is the probability that the person is an Art & Sciences student who is a second child (or more)?

P(Arts and Science and second child) = 23/223

b) Among the Art & Sciences students, what is the probability that a student is a second child (or more) ?

P(second child | Arts and Science) = 23/57

c) Among second children (or more), what is the probability that the student is enrolled in Art & Sciences ?

P(Arts and Science | second child) = 23/110

d) What is the probability that a first or only child is enrolled in the Agricultural College ?

P(Agriculture | first born) = 52/113

e) What is the probability that an Agriculture student is a first or only child ?

P(first born | Agriculture) = 52/93

f) Are enrolling in Agriculture and Human Ecology disjoint ? Explain.

Yes, since the events share no outcomes. Students can enroll in only one college.

g) Are enrolling in Agriculture and Human Ecology independent ? Explain.

No, since knowing that one event is true drastically changes the probability of the other. The probability of a student being in the Agriculture faculty is nearly 42%. The probability of a student being in the Human Ecology faculty, given that he or she is in the Agriculture faculty, is 0.

h) Are being first-born and enrolling in Human Ecology disjoint ? Explain.

No, since they share outcomes. 15 students were first-born, Human Ecology students.

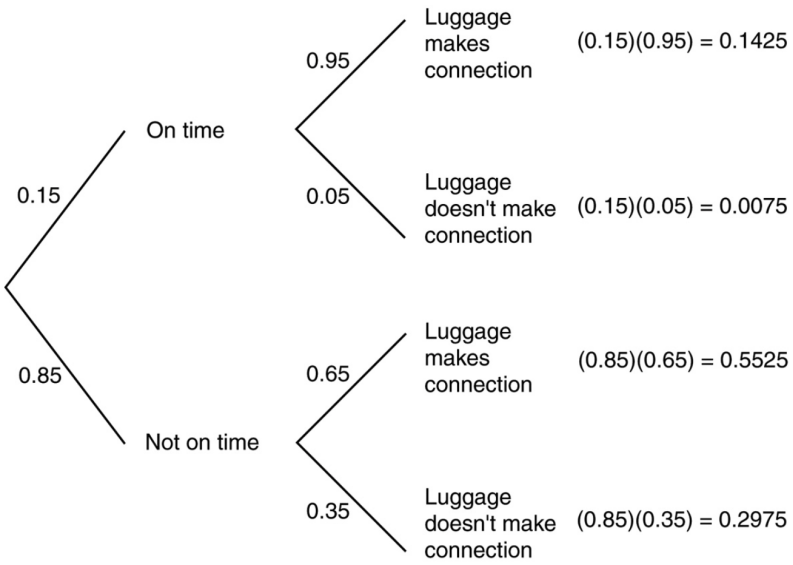
i) Are being first-born and enrolling in Human Ecology independent ? Explain.

No, since knowing that one event is true drastically changes the probability of the other. Over 19% of all students enrolled in Human Ecology, but only 13% of first-borns did.

Question 4.

Elizabeth if flying from Moncton to Vancouver with a connection in Montreal. The probability that her first flight leaves on time is 0.15. If the flight is on time, the probability that her luggage will make the connecting flight in Montreal is 0.95, but if the first flight is delayed, the probability that the luggage will make it is only 0.65.

a) Draw a "tree diagram" and organize all the information in the diagram.



b) Are the first flight and leaving on time and the luggage making the connection independent events? Explain.

No, the flight leaving on time and the luggage making the connection are not independent events. The probability that the luggage makes the connection is dependent on whether or not the flight is on time. The probability is 0.95 if the flight is on time, and only 0.65 if it is not on time.

c) What is the probability that Elizabeth's luggage arrives in Vancouver with her ?

$$\begin{aligned} P(luggage) &= P(on\ time \cap luggage) + P(not\ on\ time \cap luggage) \\ &= (0.15)(0.95) + (0.85)(0.65) = 0.695 \end{aligned}$$

d) Suppose that her dad picks her up at the Vancouver airport and her luggage is not there. What is the probability that Elizabeth's first flight was delayed ?

Using the Bay's theorem and decision tree :

$$P(not\ on\ time \mid no\ luggage) = \frac{P(not\ on\ time \cap\ no\ luggage)}{P(noluggage)} = \frac{(0.85)(0.35)}{(0.15)(0.05) + (0.85)(0.35)} \simeq 0.975$$