Concordia University
Department of Electrical and Computer Engineering
ENGR 371 - Probability and Statistics

Instructor: Dr. A. Ghrayeb | Final Exam | Summer 2003 - 20/8/2003

==

1) **(15 Marks)** Give a precise, concise definition of the following:
   a) An event.
   b) A random variable.
   c) A population.
   d) A confidence interval.
   e) Statistical inference.

2) **(15 Marks)** Let $X_1, X_2, \ldots, X_n$ be independent, normally distributed n random variables with means μ_1, μ_2, …, μ_n and variances $\sigma_1^2, \sigma_2^2, , \ldots, \sigma_n^2$, respectively. Let Y be another random variable defined as

$$Y = \sum_{i=1}^{n} X_i$$

Find the probability distribution function of Y, $g(y)$, using the notion of *moment-generating functions*. (*Show all work.*) You are given that the moment-generating function of a random variable X with mean μ and variance σ^2 is given by

$$M_X(t) = e^{j\mu t - \sigma^2 t^2/2}.$$

3) **(15 Marks)** Let X and Y be zero-mean, unit-variance independent normal random variables. Find the joint probability distribution function of V and W defined by

$$V = \sqrt{X^2 + Y^2}$$
$$W = \angle(X, Y),$$

where $\angle\theta$ denotes the angle in the range $(0, 2\pi)$ that is defined by the point (x, y).

4) **(15 Marks)** The number of automobiles that arrive at a certain intersection per minute has a Poisson distribution with mean 5. Our interest is in the time that elapses before 10 automobiles appear at the intersection.
   a) What is the probability that more than 10 automobiles appear at the intersection during any given minute of time?
   b) What is the probability that more than 2 minutes are required before 10 cars arrive?

5) **(20 Marks)** A random sample of 100 automobile owners shows that, in the province of Quebec, an automobile is driven on the average 23,500 kilometers per year with a standard deviation of 3900 kilometers.
   a) Construct a 99% confidence interval for the average number of kilometers an automobile is driven annually in Quebec.
   b) What can we assert with 99% confidence about the possible size of our error if we estimate the average number of kilometers driven by car in Quebec to be 23,500 kilometers per year?

6) **(20 Marks)** Consider a digital communication system whose model is given in the figure shown below. As shown in the figure, the system consists of three parts: the transmitter, channel, and receiver. The transmitted signal, denoted by X, is a uniform random variable that takes on the values -1 and $+1$ with equal probability. (One value is transmitted at a time.) During transmission, the channel distorts the transmitted signal by adding a noise sample to it. The added noise is a random variable, denoted by N, that is normally distributed with mean 4 and variance 0.01. The received signal, denoted by Y, is basically the algebraic sum of X and N, i.e., $Y = X + N$, which is obviously a random variable. Y is then processed, according to the decision rule given on the figure, in order to make an estimate of the transmitted signal, where the estimate is denoted by $\hat{X}$.

a) Find and sketch the conditional distribution $p(Y|X = +1)$.
b) Find and sketch the conditional distribution $p(Y|X = -1)$.
c) Find the average probability of error, i.e., when $\hat{X} \neq X$.