# MATH 135 F 2012: Assignment 9

Due: 8:30 AM, Wed., 2012 Nov. 21 in the dropboxes outside MC 4066
or in the electronic dropbox for students in the online section

Write your answers in the space provided. If you wish to typeset your solutions, use one of the solution templates posted on the course web site.

---

Family Name:

First Name:

I.D. Number:

Section:

Mark: (For the marker only.)

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

---

1. Suppose $S$ is a non-empty finite set with $k$ elements and $a \notin S$. Find a bijection $f$ from $S \cup \{a\}$ to $\mathbb{N}_{k+1}$. You do not need to prove that $f$ is a bijection, simply state it. Use this bijection to prove that $|S \cup \{a\}| = |S| + 1$.

2. The **power set** of a set $S$, denoted by $\mathcal{P}(S)$ is the set of all subsets of $S$, including the empty set and the set $S$ itself.

   (a) Let $S = \{1, 2, 3\}$. Explicitly list the elements of $\mathcal{P}(S)$.

   (b) Prove the following proposition.

   **Proposition 1.** *If $S$ is a finite set, then $|\mathcal{P}(S)| = 2^{|S|}$.*

(c) Prove the following proposition.

**Proposition 2.** *If $S$ is a set, then* $|S| \neq |\mathcal{P}(S)|$.

(Hint: Let $f : S \to \mathcal{P}(S)$ be a bijection and let $T = \{x \in S \mid x \notin f(x)\}$.)

3. Let $(a, b)$ denote the real interval $a < x < b$. Prove the following proposition.

   **Proposition 3.** $|(0,1)| = |(1,\infty)|$

4. Prove the following proposition.

   **Proposition 4.** $|\mathbb{N} \times \mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$

   (Hint: Proposition 1 of Chapter 34 on page 238 of the course notes.)

5. Let $z = 3 + i$, $u = 4 - 2i$ and $w = -3i$. Express each of the following in standard form.

(a) $z + 3u - wi$

(b) $zu\overline{w}$

(c) $z/u$

6. Express your answers to the following questions in standard form.

(a) Use the quadratic formula to find solutions to $2x^2 + 3x + 2 = 0$

(b) Use the quadratic formula to find solutions to $ix^2 + 3x - 2i = 0$

(c) The equation $x^3 - x^2 + x - 1 = 0$ has one real solution but it also has two non-real complex solutions. What are the three solutions? Show your work.

7. Use the Binomial Theorem to expand $(2ix - i)^3$.