MATH 135 F 2012: Assignment 11

DO NOT SUBMIT

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1. Find all of the cube roots of 2. Write them in standard form and plot the solutions in the complex plane.

2. Find all of the fourth roots of 2. Write them in standard form and plot the solutions in the complex plane.

3. Solve $z^2 = \dfrac{1+i}{1-i}$.

4. Solve $z^6 - z^3 - 2 = 0$. (Hint: factor the left side of the equation into a product of two cubic polynomials first. Think of it as a quadratic in z^3.)

5. Find a real cubic polynomial with roots 1 and i.

6. For each of the following polynomials $f(x) \in \mathbb{F}[x]$,

   (i) Find all of the roots in the given field $\mathbb{F}$.

   (ii) Factor $f(x)$ into factors with degree as small as possible over $\mathbb{F}[x]$.

   (a) $x^4 - 4 \in \mathbb{Q}[x]$

   (b) $x^4 - 4 \in \mathbb{R}[x]$

   (c) $x^4 - 4 \in \mathbb{C}[x]$

   (d) $x^4 - [4] \in \mathbb{Z}_5[x]$

   (e) $x^4 - a^4 \in \mathbb{F}[x]$ where $a \in \mathbb{F}$, $a \neq 0$

7. Factor $3x^4 + 13x^3 + 16x^2 + 7x + 1$ into a product of real linear factors.

8. Factor $x^5 + 1$ as far as possible into real factors.

9. Prove the following proposition.

   Proposition 1. *If $f(x)$ is a complex polynomial of degree n, then $f(x)$ has n roots.*

10. Prove the following theorem.

    Theorem 1 (Conjugate Roots Theorem (CJRT))**.** *If $c \in \mathbb{C}$ is a root of the real polynomial $f(x)$, then $\bar{c} \in \mathbb{C}$ is a root of $f(x)$.*

11. Prove the following proposition.

    Proposition 2 (Factoring Real Polynomials (FRP))**.** *Real polynomials can be factored into products of real linear and real quadratic factors.*

12. Prove the following theorem.

    Theorem 2 (Remainder Theorem (RT))**.** *The remainder when the polynomial $f(x)$ is divided by $(x - c)$ is $f(c)$.*