MATH 135 F 2012: Assignment 10

Due: 8:30 AM, Wed., 2012 Nov. 28 in the dropboxes outside MC 4066
or in the electronic dropbox for students in the online section

Write your answers in the space provided. If you wish to typeset your solutions, use one of the solution templates posted on the course web site.

Family Name:

First Name:

I.D. Number:

Section:

Mark: (For the marker only.)

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

---

1. (a) Write $z = \dfrac{9+i}{5-4i}$ in the form $r(\cos\theta + i\sin\theta)$ with $r \geq 0$ and $0 \leq \theta < 2\pi$.

   (b) Write $z = \left(\dfrac{e^{i5\pi/12}}{\sqrt{3}}\right)^{-6}$ in the form $x + iy$ with $x, y \in \mathbb{R}$.

2. Compute $(2-2\sqrt{3}i)^4$ twice: once using the Binomial Theorem and once using De Moivre's Theorem. Write your answer in standard form.

3. Write an expression for $(\cos\theta + i\sin\theta)^3$ using the Binomial Theorem and De Moivre's Theorem. Equate real and imaginary parts to show

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

4. Let $z \in \mathbb{C}$. Prove that $\bar{z}z = |z|^2$.

5. Suppose a, b and z are complex numbers. Prove that the equation

$$|z - a| = |z - b|$$

implies that there exists a complex number p and a real number n so that

$$\bar{p}z + p\bar{z} = n$$

6. (a) Find the cube roots of -1. Express your answers in standard form and plot these solutions in the complex plane.

(b) Find the cube roots of $-i$. Express your answers in standard form and plot these solutions in the complex plane.

(c) Find all solutions to $z^6 + (i+1)z^3 + i = 0$. Express your answers in standard form.

7. Let $z \in \mathbb{C}$, $z \neq \pm i$. Prove that $\dfrac{z}{1+z^2}$ is real if and only if z is real or $|z| = 1$.

8. For each step of the following argument, provide justification (by citing an appropriate proposition, for example) or state that the logic is incorrect and give a reason why.

Let z be any complex number.

$$\begin{aligned}
0 &= |1| - 1 && (1)\\
&= |zz^{-1}| - 1 && (2)\\
&= |z||z^{-1}| - 1 && (3)\\
&= |(z+1) - 1||z^{-1}| - 1 && (4)\\
&= (|z+1| + |-1|)|z^{-1}| - 1 && (5)\\
&= (|z+1| + 1)|z^{-1}| - 1 && (6)\\
&= |z+1||z^{-1}| + |z^{-1}| - 1 && (7)\\
&= |z+1||z| + |z| - 1 && (8)\\
&= (|z| + |1|)|z| + |z| - 1 && (9)\\
&= (|z| + 1)|z| + |z| - 1 && (10)\\
&= |z|^2 + 2|z| - 1 && (11)\\
&= (|z| - 1)^2 && (12)
\end{aligned}$$

Therefore $|z| = 1$ for all complex numbers z.

Analysis

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

(12)

(a) Concluding statement.