MATH 135 F 2012: Assignment 7

Due: 8:30 AM, Wed., 2012 Nov. 7 in the dropboxes outside MC 4066
or in the electronic dropbox for students in the online section

Write your answers in the space provided. If you wish to typeset your solutions, use one of the solution templates posted on the course web site.

Family Name:

First Name:

I.D. Number:

Section:

Mark: (For the marker only.)

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

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1. For each of the following, determine the complete solution, if a solution exists.

   (a) $4x \equiv 7 \pmod{20}$

   (b) $[3][x] = [11]$ in $\mathbb{Z}_{21}$

   (c) $21x \equiv 9 \pmod{117}$

2. This question asks you to carefully examine the properties of linear congruences.

   (a) Find integers $c \neq 0, a, b, m$ such that the solution set of $ax \equiv b \pmod{m}$ is different from the solution set of $cax \equiv cb \pmod{m}$.

   (b) Suppose we want a proposition that says

   Proposition 1. *If ________________, then $ax \equiv b \pmod{m}$ and $cax \equiv cb \pmod{m}$ have the same set of solutions.*

   Determine a simple condition on c and m for the hypothesis that makes this proposition correct, and prove this proposition.

3. Solve each of the following systems of equations.

(a) $$\begin{aligned} x &\equiv 5 \pmod 9 \\ x &\equiv 3 \pmod 7 \end{aligned}$$

(b) $$\begin{aligned} 3x &\equiv 2 \pmod 8 \\ 4x &\equiv 9 \pmod{11} \end{aligned}$$

(c) In $\mathbb{Z}_{11}$, $$\begin{aligned} [2][x] + [7][y] &= [4] \\ [x] + [2][y] &= [9] \end{aligned}$$

4. Consider the following two systems of linear congruences:

$$A : \begin{cases} n \equiv 2 \pmod{12} \\ n \equiv 10 \pmod{18} \end{cases} \qquad B : \begin{cases} n \equiv 5 \pmod{12} \\ n \equiv 11 \pmod{18} \end{cases}$$

(a) Determine which one has solutions and which one has no solutions. For the one with solutions, give the complete solutions to the system. For the one with no solutions, explain why no solutions exist.

(b) Let a_1, a_2 be integers, and let m_1, m_2 be positive integers. Consider the following system of linear congruences

$$S : \begin{cases} n \equiv & a_1 \pmod{m_1} \\ n \equiv & a_2 \pmod{m_2} \end{cases}$$

Based on your observations in part (a), complete the following two propositions.

Proposition 2. *The system S has a solution if and only if* ____________________________.

(This blank needs to be filled with a simple condition on a_1, a_2, m_1, m_2.)

Proposition 3. *If n_0 is a solution to S, then the complete solution is*

$$n \equiv \underline{\hspace{3cm}}.$$

(c) Prove Proposition 2.

5. Solve $49x^{177} + 37x^{26} + 3x^2 + x + 1 \equiv 0 \pmod{7}$.

6. Solve $2x^{121} + 22x^{36} + 21x^{30} + 2 \equiv 0 \pmod{77}$. (From Winter 2012 Final Examination.)