MATH 135 F 2012: Assignment 6

Due: 8:30 AM, Wed., 2012 Oct. 31 in the dropboxes outside MC 4066
or in the electronic dropbox for students in the online section

Write your answers in the space provided. If you wish to typeset your solutions, use one of the solution templates posted on the course web site.

Family Name:

First Name:

I.D. Number:

Section:

Mark: (For the marker only.)

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

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1. In each of the following cases, find all values of a, $0 \le a < m$ where m is the modulus, that satisfy the congruence relation. You do not need to show your work.

   (a) $a \equiv 52 \pmod{12}$

   (b) $-14 \equiv a \pmod{15}$

   (c) $2a \equiv 5 \pmod{7}$

   (d) $2a \equiv 5 \pmod{8}$

   (e) $2a \equiv 4 \pmod{8}$

   (f) $a^2 \equiv 1 \pmod{7}$

2. Construct addition and multiplication tables for $\mathbb{Z}_6$. Which elements of $\mathbb{Z}_6$ have multiplicative inverses?

3. In each of the following cases, find all values of $[x] \in \mathbb{Z}_m$, $0 \leq x < m$, that satisfy the equation. You do not need to show your work.

   (a) $[4][3] + [5] = [x] \in \mathbb{Z}_{10}$

   (b) $[7]^{-1} - [2] = [x] \in \mathbb{Z}_{10}$

   (c) $[2][x] = [4] \in \mathbb{Z}_8$

   (d) $[3][x] = [9] \in \mathbb{Z}_{11}$

4. What is the remainder when 3141^{2001} is divided by 17?

5. This question deals with divisibility by nine.

   (a) Let $n = 124578$ and d be the sum of the digits of n.

      i. Determine the value of d.

      ii. Does $9 \mid d$?

      iii. Does $9 \mid n$?

   (b) Let $n = 27182818$ and d be the sum of the digits of n.

      i. Determine the value of d.

      ii. Does $9 \mid d$?

      iii. Does $9 \mid n$?

   (c) Prove the following proposition.

   Proposition 1. *Let n be a positive integer and let d be the sum of the digits of n. Then n is divisible by 9 if and only if d is divisible by 9.*

   Hint: Let the decimal representation of n be $a_r a_{r-1} a_{r-2} \dots a_1 a_0$. Then

$$n = 10^r a_r + 10^{r-1} a_{r-1} + 10^{r-2} a_{r-2} + \dots + 10 a_1 + a_0$$

6. Let $a, b, c \in \mathbb{Z}$. Consider the following statement:

Statement 2. *For every integer x_0, there exists an integer y_0 such that $ax_0 + by_0 = c$.*

(a) Determine conditions on a, b, c such that Statement 2 is true if and only if these conditions hold. State and prove this if and only if statement.

(b) Using part (a), write down one set of values for a, b, c for which Statement 2 is false.

(c) Write down the negation of Statement 2 without using any form of the word "not" (the symbol $\neq$ is acceptable). We will call your answer **Statement 3**.

(d) Prove that Statement 3 is true for the set of values you have chosen in part (b).

7. (a) Prove that: if $a \mid c$ and $b \mid c$ and $\gcd(a, b) = 1$, then $ab \mid c$.

(b) Show that $\gcd(a, b) = 1$ is a necessary condition for the preceding statement to be true. That is, show that the following statement is false: If $a \mid c$ and $b \mid c$, then $ab \mid c$.

(c) Using (a), prove that: For all integers n, $21 \mid (3n^7 + 7n^3 + 11n)$.