MATH 135 F 2012: Assignment 4

Due: 8:30 AM, Wed., 2012 Oct. 10 in the dropboxes outside MC 4066
or in the electronic dropbox for students in the online section

Write your answers in the space provided. If you wish to typeset your solutions, use one of the solution templates posted on the course web site.

Family Name:

First Name:

I.D. Number:

Section:

Mark: (For the marker only.)

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

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1. Expand and simplify $\left(3x + \frac{y}{2}\right)^4$. Use the Binomial Theorem.

2. What is the coefficient of x^{25} in the expansion of $\left(2x^4 - \frac{3}{x^3}\right)^{15}$?

3. What is wrong with the following "inductive proof" that $n = 1$ for all integers $n \geq 1$.

Inductive Statement: $P(n)$: $n = 1$.
Basis Case: We show that $P(1)$ is true. $1 = 1$, so the statement is true for $n = 1$.
Inductive Hypothesis: We assume that $P(i)$ is true for $1 \leq i \leq k$. That is, $i = 1$ for all i in the range $1 \leq i \leq k$.
Inductive Conclusion: We show that $P(k + 1)$ is true. Note that

$$k + 1 = k + k - (k - 1)$$

and since, by the inductive hypothesis, $k = 1$ and $k - 1 = 1$

$$k + 1 = k + k - (k - 1) = 1 + 1 - (1) = 1$$

as required.

4. Prove that
$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots+\frac{1}{n^2}<2-\frac{1}{n}$$
for every positive integer $n>1$.

5. For each of the following statements, either prove the statement or disprove it using a counterexample.

   (a) Let $a, b \in \mathbb{Z}$. If $a \mid b^2$, then $a \mid b$.

   (b) For any integer a, $\gcd(22a + 7, 3a + 1) = 1$.

   (c) If r is irrational, then $1/r$ is irrational.

   (d) For any integer a, $\gcd(a^2, a + 1) = 1$.

6. Let $a, b, c \in \mathbb{Z}$. If $c > 0$, then $\gcd(ac, bc) = c \gcd(a, b)$. (Suggestion: Let $d = \gcd(a, b)$. Show $cd = \gcd(ac, bc)$.)

7. (For practice only. A question like this may appear on the midterm with smaller numbers.) Using the Euclidean Algorithm or the Extended Euclidean Algorithm, compute $d = \gcd(42042, 1071)$. Certify that your value of d is correct by showing that the conditions of the GCD Characterization Theorem are satisfied.