MATH 135 F 2012: Assignment 2

Due: 8:30 AM, Wed., 2012 Sep. 26 in the dropboxes outside MC 4066
or in the electronic dropbox for students in the online section

Write your answers in the space provided. If you wish to typeset your solutions, use one of the solution templates posted on the course web site.

Family Name:

First Name:

I.D. Number:

Section:

Mark: (For the marker only.)

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

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1. A **tautology** is a logical statement that is true for all possible values of the component statements. Use a truth table to determine whether or not

$$(A \Rightarrow (B \vee C)) \iff ((A \wedge \neg B) \Rightarrow C)$$

is a tautology.

A	B	C	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

2. Are the following pieces of reasoning valid? (Does the third statement follow from the first two?) Explain.

   (a) If I do every problem in the text book, then I will learn discrete mathematics.
   I learnt discrete mathematics.
   Therefore, I must have done every problem in the text book.

   (b) If I do every problem in the text book, then I will learn discrete mathematics.
   I did not learn discrete mathematics.
   Therefore, there is a problem in the book which I did not do.

   (c) If I miss a lecture, then I am ill or I had a late night.
   I had a late night.
   Therefore, I missed a lecture.

3. An integer $p > 1$ is called a **prime** if its only divisors are 1 and p. Otherwise, p is called **composite**. Consider the proposition below.

   Proposition 1. *For every prime number p, $p + 7$ is composite.*

   (a) Identify the four parts of the quantified statement.

   Quantifier:
   Variable:
   Domain:
   Open sentence:

   (b) Prove the proposition.

4. Consider the proposition below.

Proposition 2. *There exists an integer k so that $x^2 + kx + 2 = 0$ has an integer solution.*

(a) Identify the four parts of the quantified statement with variable k.

Quantifier:
Variable: k
Domain:
Open sentence:

(b) Prove the proposition.

5. Consider the proposition below.

Proposition 3. *For all numbers $a, b \in \mathbb{R}$, $\min\{a, b\} \leq (a + b)/2$.*

(a) Identify the four parts of the quantified statement.

Quantifier:
Variable:
Domain:
Open sentence:

(b) Prove the proposition.

6. Prove the proposition below.

   Proposition 4. *For all real numbers* $\varepsilon > 0$ *and* $a > 0$, *there exists an integer* $n > 0$ *such that* $\dfrac{a}{n} < \varepsilon$.

7. Prove that the function $f : (0, \infty) \to (0, \infty)$ defined by $f(x) = \ln x^2$ is surjective.

8. For this question you need to read about the **floor** function on page 66 and **injective** (or **one-to-one**) functions in Section 31.2.1 on pages 219 – 220. Consider the following statement.

Statement 5. *Let* $f : \mathbb{Z} \to \mathbb{Z}$ *be the function defined by* $f(x) = \lfloor x/2 \rfloor$. *Then* f *is injective.*

The following is a proposed proof written out line by line.

1 We will show that f is injective. That is, we will show that for all $a, b \in \mathbb{Z}$, if $f(a) = f(b)$ then $a = b$.

2 From the hypothesis, let $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$ be such that $f(a) = f(b)$.

3 It suffices to show that $a = b$. We will proceed by the Direct Method: assume the hypothesis and deduce the conclusion.

4 From (2) we see that $\lfloor a/2 \rfloor = \lfloor b/2 \rfloor$.

5 From (4) we see that $2\lfloor a/2 \rfloor = 2\lfloor b/2 \rfloor$.

6 From (5) we see that $\lfloor 2a/2 \rfloor = \lfloor 2b/2 \rfloor$.

7 From (6) we see that $\lfloor a \rfloor = \lfloor b \rfloor$.

8 Since $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$, from (7) we see that $a = b$.

9 From (2) and (8) it follows that f is injective.

(a) The proposed proof is not correct. Identify all of the errors.

(b) Is the function f injective? If it is, give a proof. If not, give a counter-example.