

MATH 135 F 2012: Assignment 1

Due: 8:30 AM, Wed., 2012 Sep. 19 in the dropboxes outside MC 4066
or in the electronic dropbox for students in the online section

Write your answers in the space provided. If you wish to typeset your solutions, use one of the solution templates posted on the course web site.

Family Name:

First Name:

I.D. Number:

Section:

Mark: (For the marker only.)

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

1. Use a truth table to determine whether or not $\neg(A \Rightarrow B) \equiv A \wedge (\neg B)$.

A	B	$A \Rightarrow B$	$\neg(A \Rightarrow B)$	$\neg B$	$A \wedge (\neg B)$
T	T				
T	F				
F	T				
F	F				

2. Use a truth table to determine whether or not $A \vee (B \wedge C)$ is equivalent to $(A \vee B) \wedge (A \vee C)$.

A	B	C	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

3. This question deals with sets.

(a) Give a specific example to show that the statement " $U \cup (S \cap T) = (U \cup S) \cap T$ " is false.

(b) Complete the proof of the following proposition.

Proposition 1. *Let S, T, U be sets. Then $U \cup (S \cap T) = (U \cup S) \cap (U \cup T)$.*

$$x \in U \cup (S \cap T) \iff (x \in U) \vee (x \in S \cap T) \quad (\text{definition of union}) \quad (1)$$

$$\iff \quad (2)$$

4. An integer n is **even** if $2 \mid n$. Otherwise, the integer is **odd**. This question deals with the following proposition.

Proposition 2. *If m and n are even integers, then $m + n$ is an even integer.*

- (a) State the hypothesis of Proposition 2.
- (b) State the conclusion of Proposition 2.
- (c) Prove the proposition.

5. This question deals with the following proposition.

Proposition 3. *Let a, b, c and d be integers. If $a \mid c$ and $b \mid d$, then $ab \mid cd$.*

- (a) State the hypothesis of Proposition 3.
- (b) State the conclusion of Proposition 3.
- (c) Prove the proposition.

6. This question deals with the following proposition.

Proposition 4. *Let a and b be integers. If $a \mid b$ and $b \mid a$, then $a = \pm b$.*

- (a) State the hypothesis of Proposition 4.
- (b) State the conclusion of Proposition 4.
- (c) Prove the proposition.

7. Suppose a is an integer. Consider the following statement.

Statement 5. *If $32 \nmid ((a^2 + 3)(a^2 + 7))$, then a is even.*

In trying to prove or disprove this statement, each of the proposed proofs in parts (a) and (b) contains a flaw. Determine the main flaw in each argument.

- (a) Suppose a is even. Then a^2 is even, so both $a^2 + 3$ and $a^2 + 7$ are odd. Since 32 is even, $32 \nmid ((a^2 + 3)(a^2 + 7))$.
- (b) Suppose $32 \nmid ((a^2 + 3)(a^2 + 7))$. Since $2 \mid 32$, $2 \nmid ((a^2 + 3)(a^2 + 7))$. This means that $(a^2 + 3)(a^2 + 7)$ must be odd, so both $a^2 + 3$ and $a^2 + 7$ must be odd. Therefore, a^2 is even, and hence a is even.
- (c) State the contrapositive of Statement 5.
- (d) Prove Statement 5.