NAME: Solutions STUDENT ID# :

LAKEHEAD UNIVERSITY

FACULTY OF ENGINEERING

Midterm Exam -2015

Engineering	**3021**	**AA**	**Summer Transition Program**
SUBJECT	COURSE NO.	SECTION	

Engineering Analysis A	**Dr. M. Nasir Uddin**
COURSE TITLE	INSTRUCTOR

Wed, July 22, 2015	**9:00 a.m. - 11 a.m.**	**2 hours**	**AT1003/UC2011**
EXAM DATE	EXAM TIME	DURATION	ROOM

- **NO CALCULATOR OR ELECTRONIC GADGETS ALLOWED**
- **SHOW YOUR STEPS – NO STEPS NO MARKS**
- **CLOSED BOOK EXAM**

This exam **MAY NOT** be removed from the room

STUDENTS PLEASE NOTE

YOU MUST count the number of pages in this examination question paper **BEFORE** beginning to write, and report any discrepancy immediately to a proctor. **MARKS** are indicated in the right hand margin.

This is page 1 of 11.

This section is for faculty use only

Q	1	2	3	4	5	6	7	8.a	8.b	9	10	Total
M												

1. Solve the inequality $x^2 - 3x - 18 \leq 0$ in terms of intervals and illustrate the solution set on the real number line. 10

Sol^n –

4

$x^2 - 3x - 18 \leq 0$

$x^2 - 6x + 3x - 18 \leq 0$

$(x-6)(x+3) \leq 0$

If, $(x-6)(x+3) = 0$ then, $x = 6, -3$

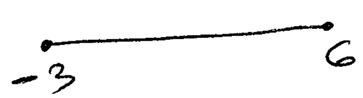
4

	$(x-6)$	$(x+3)$	$(x-6)(x+3)$
$x < -3$	$(-)$	$(-)$	$(+)$
$-3 < x < 6$	$(-)$	$(+)$	$(-)$ ✓
$x > 6$	$(+)$	$(+)$	$(+)$

2

$\therefore$ The solution is, $-3 \leq x \leq 6$

$\therefore x = [-3, 6]$



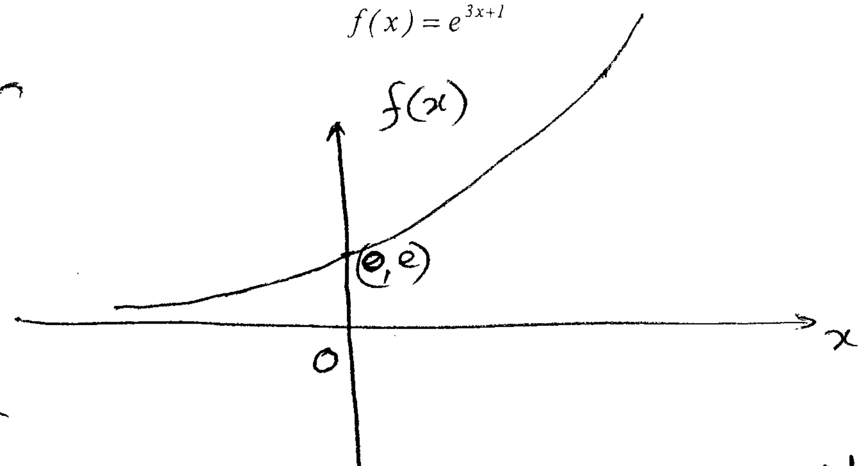
2. Sketch the graph of the following function and determine whether it is one-to-one. If yes, find a formula for the inverse of the function.

10

$$f(x) = e^{3x+1}$$

Sol^n –

2



2 It is a one-to-one function as it passes the horizontal line test (i.e., no horizontal line intersects the curve more than once).

4

$$y = f(x) = e^{3x+1}$$

$$\therefore \ln y = \ln e^{(3x+1)} = 3x+1$$

$$\Rightarrow x = \frac{\ln y}{3} - \frac{1}{3}$$

2

Interchanging x & y,

$$y = \frac{\ln x}{3} - \frac{1}{3} = f^{-1}(x)$$

3. Find the composite functions $f \circ g$, $g \circ f$, $g \circ g$ and their domains. (10)

$$f(x) = \sqrt{2x-1}, \quad g(x) = x^2 - 1$$

Sol^n – $f \circ g = f(g(x)) = \sqrt{2g(x) - 1} = \sqrt{2x^2 - 2 - 1} = \sqrt{2x^2 - 3}$

Domain of $f \circ g$: $2x^2 - 3 \geq 0 \Rightarrow x^2 \geq 3/2 \Rightarrow |x| \geq \sqrt{3/2}$

$\therefore x \leq -\sqrt{3/2}$ and $x \geq \sqrt{3/2}$

$g \circ f = g(f(x)) = (f(x))^2 - 1 = 2x - 1 - 1 = 2x - 1$

Domain of $g \circ f$: $D = \mathbb{R} \mid$ (x is in domain of 'f' and 'f' is in domain of 'g')

$\therefore D = \mathbb{R} \mid x \geq 1/2 = [\tfrac{1}{2}, \infty) = \mathbb{R} \mid$ ($x \geq 1/2$ and $x > \tfrac{1}{2} \in \mathbb{R}$)

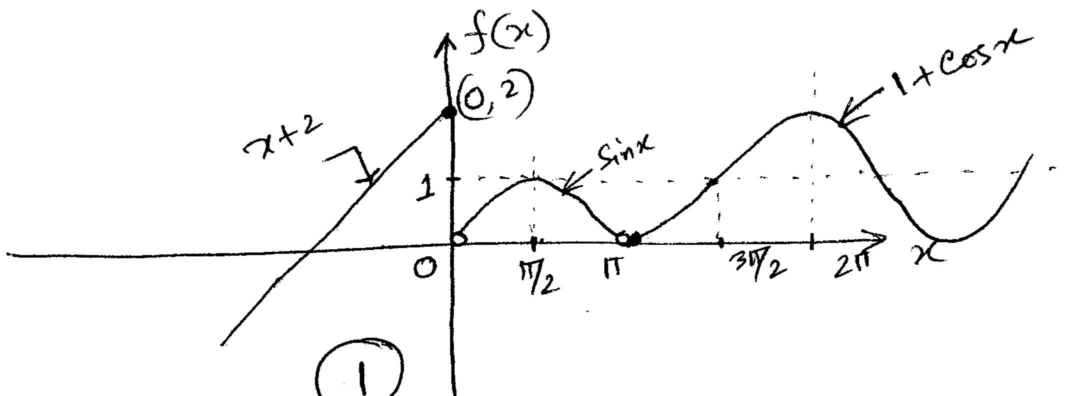
$g \circ g = g(g(x)) = (g(x))^2 - 1 = (x^2 - 1)^2 - 1 = x^4 - 2x^2 + 1 - 1$
$= x^4 - 2x^2$

Domain of $g \circ g$: $D = \mathbb{R}$

4. Find $\lim_{x\to 0^-} f(x), \lim_{x\to 0^+} f(x), \lim_{x\to 0} f(x), f(0), \lim_{x\to \pi^-} f(x), \lim_{x\to \pi^+} f(x), and\ f(\pi)$ and hence determine at which points the function is continous. If not continutous, is it it continuous from left, right or neither side? Explain why by using the definition of continuty.

10

$$f(x) = \begin{cases} 2+x & if \quad x \le 0 \\ sin\, x & if \quad 0 < x < \pi \\ 1+cos\, x & if \quad x \ge \pi \end{cases}$$



1 $\lim_{x\to 0^-} f(x) = 2$, 1 $\lim_{x\to 0^+} f(x) = 0$

1 $\lim_{x\to 0} f(x)$ does not exist since, $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

1 $f(0) = 2$

1 $\lim_{x\to \pi^-} f(x) = 0$, 1 $\lim_{x\to \pi^+} f(x) = 0$, 1 $f(\pi) = 0$

3 The function is continuous everywhere except at $x=0$.
But $f(x)$ is continuous from left at $x=0$ since,

$$\lim_{x\to 0^-} f(x) = f(0)$$

5. Find the limit if it exists. If the limit does not exist explain why.

10

$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{x^2 + 4x - 21}$$

$x = 3$, $\frac{0}{0}$ form, undefined

Sol^n -

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x-2)}{(x+7)\cancel{(x-3)}}$$

$$= \lim_{x \to 3} \frac{x-2}{x+7}$$

$$= \frac{3-2}{3+7} = \frac{1}{10}$$

6. Find the following limits for the given function $f(x) = \dfrac{x^3 + 3x^2 - 4}{3x^3 + 2x}$. Does the function have any horizontal or vertical asymptotes? 10

$$\lim_{x \to \infty} f(x), \quad \lim_{x \to 1} f(x), \quad \lim_{x \to 0} f(x)$$

Sol^n-

3

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^3 + 3x^2 - 4}{3x^3 + 2x}$$

$$= \lim_{x \to \infty} \frac{\frac{x^3 + 3x^2 - 4}{x^3}}{\frac{3x^3 + 2x}{x^3}} = \lim_{x \to \infty} \frac{1 + \frac{3}{x} - \frac{4}{x^3}}{3 + \frac{2}{x^2}}$$

$$= \frac{1 + 0 - 0}{3 + 0} = \frac{1}{3}$$

2.5

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^3 + 3x^2 - 4}{3x^3 + 2x} = \frac{1 + 3 - 4}{3 + 2} = \frac{0}{5} = 0$$

2.5

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^3 + 3x^2 - 4}{3x^3 + 2x} = \frac{0 + 0 - 4}{0 + 0} = \infty$$

$\therefore$ $x = 0$ is the vertical asymptote.

$y = \frac{1}{3}$ is the horizontal " .

2

7. Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $y = \frac{2}{x}$. Then find the equation of the tangent line to the graph of $y = \frac{2}{x}$ at point P(2, 1). 10

Sol^n-

5

$$y = f(x) = \frac{2}{x}$$

$$\therefore \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h \to 0} \frac{\cancel{2x} - \cancel{2x} - 2\cancel{h}}{\cancel{h}\, x(x+h)}$$

$$= \frac{-2}{x(x+0)} = \frac{-2}{x^2}$$

2

$\therefore$ The slope of tangent line at (2, 1)

$$m = \frac{-2}{2^2} = \frac{-1}{2}$$

3

$\therefore$ The eq^n of tangent line,

$$y - 1 = -\frac{1}{2}(x-2) = -\frac{x}{2} + 1$$

$$\Rightarrow y + \frac{x}{2} = 2$$

8. Differentiate the following functions

(a) $f(x) = sin(\ cos\, x + 3x^3\ )$ (Use chain rule) 5

$$f'(x) = \cos(\cos x + 3x^3)\, \frac{d}{dx}(\cos x + 3x^2)$$

$$= \cos(\cos x + 3x^3)\,(-\sin x + 9x^2)$$

$$= (9x^2 - \sin x)\,\cos(\cos x + 3x^3)$$

(b) $y = ln(\sqrt{x}e^{x^2}\ )$ (Hint: *ln (ab)* = *ln(a)* + *ln(b)* and, *ln* (a^n) = *n ln(a)*) 5

$$y = \ln\sqrt{x} + \ln e^{x^2}$$

$$= \frac{1}{2}\ln x + x^2$$

$$\therefore \frac{dy}{dx} = \frac{1}{2x} + 2x$$

9. Use implicit differentiation to differentiate the following function. Then find the slope of the tangent line at the point (2, 1).

10

$$x^4 y^3 - 4xy^2 + 2y - 10 = 0$$

Sol^n- Differentiate both sides of the equation w.r. to x,

8

$$4x^3y^3 + x^4 \times 3y^2 \frac{dy}{dx} - 4y^2 - 4x \times 2y \frac{dy}{dx} + 2\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}\left(3x^4y^2 - 8xy + 2\right) = 4y^2 - 4x^3y^3$$

$$\Rightarrow \frac{dy}{dx} = \frac{4y^2 - 4x^3y^3}{3x^4y^2 - 8xy + 2}$$

$\therefore$ The slope of tangent line at (2,1)

2

$$m = \frac{4 \times 1^2 - 4 \times 2^3 \times 1^3}{3 \times 2^4 \times 1^2 - 8 \times 2 \times 1 + 2}$$

$$= \frac{4-32}{48-16+2} = \frac{-28}{34} = \frac{-14}{17}$$

10. Find the 2nd derivative, $f''(x)$ of the function, $f(x) = \dfrac{x^2 + 1}{x - 1}$ 10

(Hint: $\dfrac{d(u/v)}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$)

Sol^n –

5

$$f'(x) = \frac{(x-1)\,2x - (x^2+1)(1-0)}{(x-1)^2}$$

$$f'(x) = \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2} = \frac{x^2 - 2x - 1}{(x-1)^2}$$

5

$$f''(x) = \frac{d}{dx} f'(x) = \frac{(x-1)^2(2x-2) - (x^2-2x-1) \times 2(x-1)}{(x-1)^4}$$

$$= \frac{2(x-1)\left\{(x-1)^2 - (x^2-2x-1)\right\}}{(x-1)^4}$$

$$= \frac{2(\cancel{x^2} - \cancel{2x} + 1 - \cancel{x^2} + \cancel{2x} + 1)}{(x-1)^3}$$

$$= \frac{4}{(x-1)^3}$$