

MA121 Mock Test 2

Name: _____

**** Please remember that mock tests are meant as a means of providing an extra set of practice questions and basis for a review class. Do not study for the midterm based solely on the topics covered by the mock test! Go back through notes/assignments/homework to ensure you have reviewed all concepts discussed in the course.

Time Allowed: 80 minutes

Total Value: 65 marks

Number of Pages: 6

Instructions:

Cheat Sheet: One 8.5" × 11" page of study notes (both sides) is allowed as a reference while completing the mock test. Please note, that the cheat sheet is permitted for the mock test only!!

Use of the Casio FX-300MS Plus Calculator is permitted. No other aids allowed.

Answer in the spaces provided.

Show all your work. Insufficient justification will result in a loss of marks.

1. [9 marks]

(a) Let $A = \{3, c, \{c\}, \{c, d\}\}$. State the number of elements in the power set of B ; that is, determine $|\mathcal{P}(A)|$.

(b) Let $A = \{a, b, c, e\}$ and $B = \{1, a\}$. Determine $A \times B$.

(c) State the dual of the equation from set algebra: $(A \cup B \cup C)^c = (A \cup B)^c \cap (A \cup C)^c$

(d) Let $A = \{1, 2, \dots, 10\}$. Give an example of two sets S and B such that $S \subseteq \mathcal{P}(A)$, $|S| = 4$, $B \in S$ and $|B| = 2$.

(e) Given $|U| = 80$, $|A| = 30$, $|B| = 42$ and $|A \cap B| = 9$, determine $|A^c \cap B^c|$.

2. [12 marks] Prove or disprove each of the following. State the method of proof used.

(a) For any three sets A , B and C , $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(b) For every two sets A and B , $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

(c) If z is a complex number such that $(\bar{z})^2 = z^2$, then z is pure real or z is pure imaginary.

3. [10 marks]

(a) Suppose $n \in \mathbb{Z}$, $n \geq 2$. Show that $\binom{n+1}{2} - \binom{n}{2} = n$.

(b) Use induction to prove that $\sum_{i=1}^n i(i!) < (n+1)!$ for all $n \in \mathbb{N}$.

4. [10 marks] Use the Binomial Theorem for each of the following.

(a) Find the term containing x^{12} in the expansion of: $\left(5x^7y^3 - \frac{1}{2x^2}\right)^{12}$.

(b) Determine the value of: $\sum_{k=1}^{2014} (-1)^k \binom{2014}{k} 5^k$.

5. [10 marks] Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{5\}$.

Define $f : A \rightarrow B$ by $f(x) = \frac{5x+1}{x-2}$ for all $x \in A$.

(a) State the domain and codomain of f .

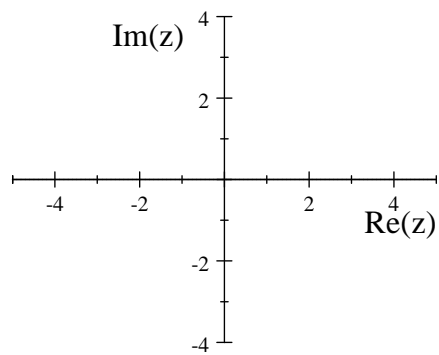
(b) Show that f is bijective (that is, that f is both one-to-one, and onto).

6. [4 marks] Given $\sin \theta = -\frac{\sqrt{3}}{2}$ and $\cos \theta > 0$, determine the value of θ , where $0 \leq \theta \leq 2\pi$.

7. [11 marks]

(a) Divide the complex numbers $\frac{i+4}{5-\sqrt{3}i}$. Express your final answer in standard form.

(b) Represent all $z \in \mathbb{C}$ in the complex plane, such that $|z| \leq 2$.



(c) Find all complex numbers such that $\bar{z} = z^2$.

(d) If z and w are any two complex numbers, show that: $|zw| = |z| |w|$.