CHAPTER

6

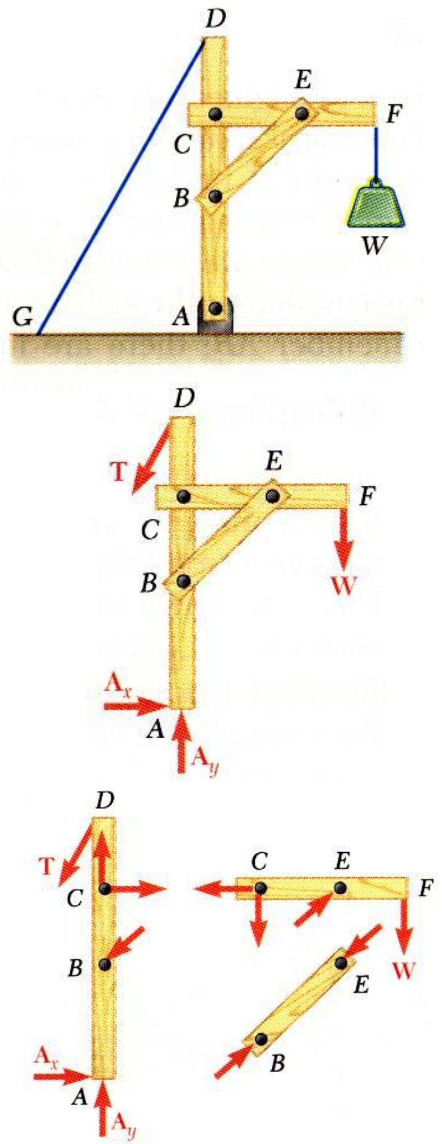
VECTOR MECHANICS FOR ENGINEERS:

STATICS

Analysis of Structures

Contents

Introduction

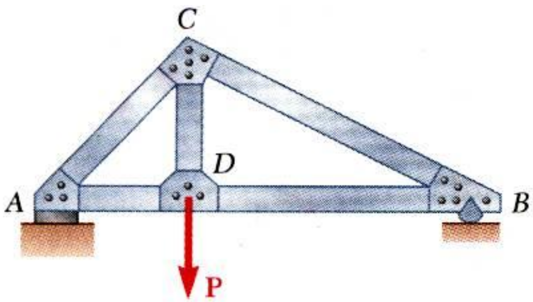


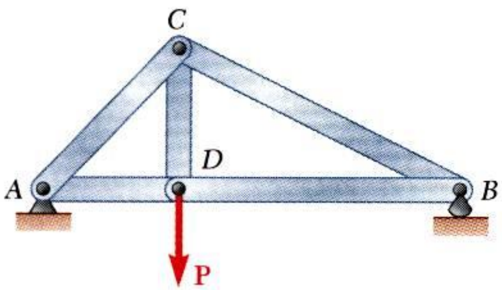
- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.

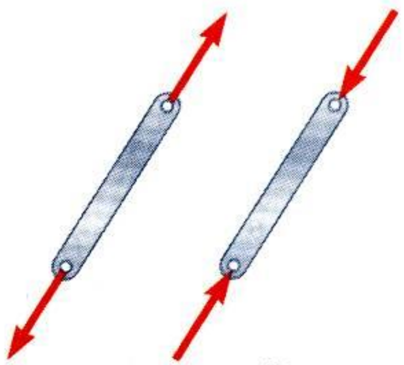
- In the interaction between connected parts, Newton's 3rd Law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite sense.

- Three categories of engineering structures are considered:
  - *a) Frames*: contain at least one one multi-force member, i.e., member acted upon by 3 or more forces.
  - *b) Trusses*: formed from *two-force members*, i.e., straight members with end point connections
  - *c) Machines*: structures containing moving parts designed to transmit and modify forces.

Definition of a Truss

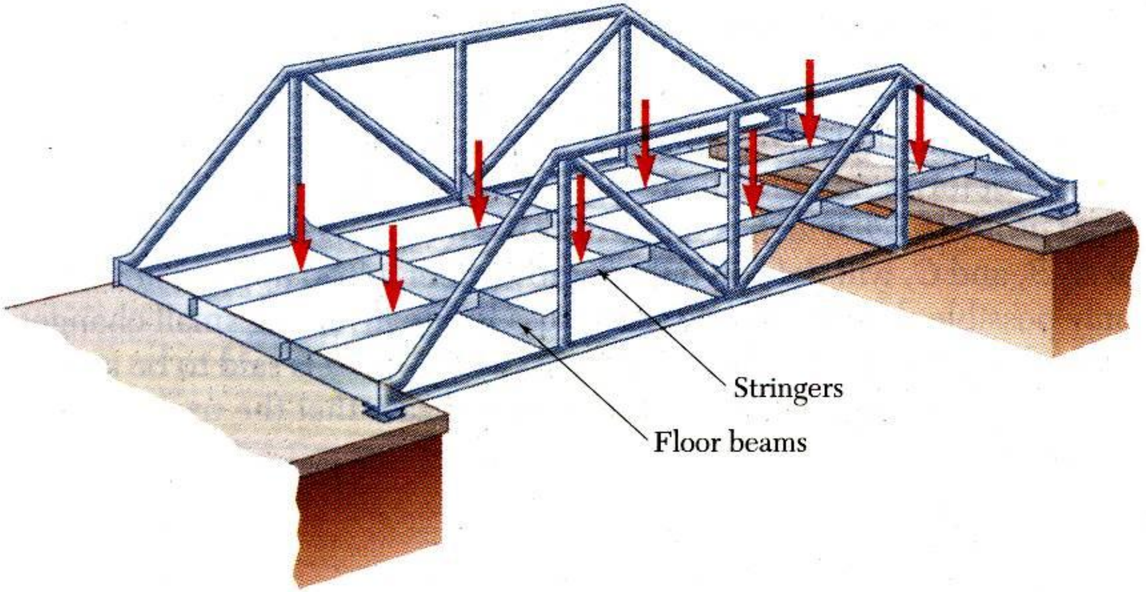






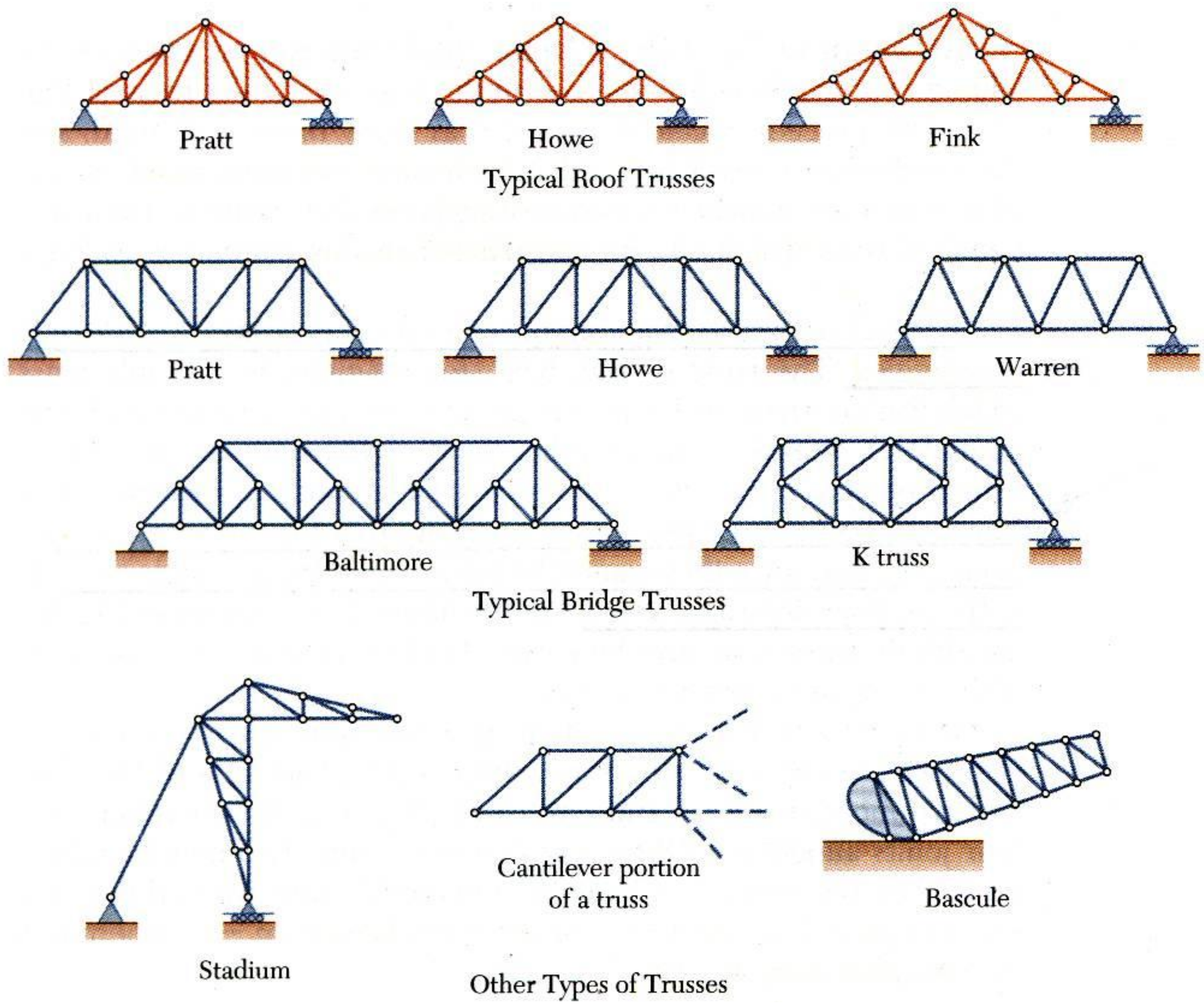
- A truss consists of straight members connected at joints. No member is continuous through a joint.
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.

Definition of a Truss

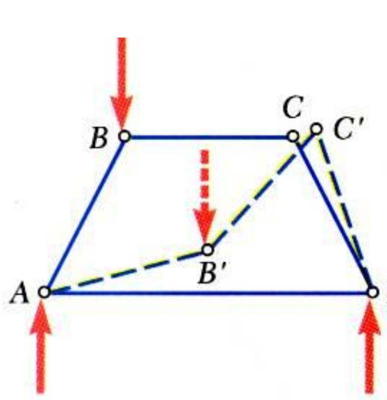


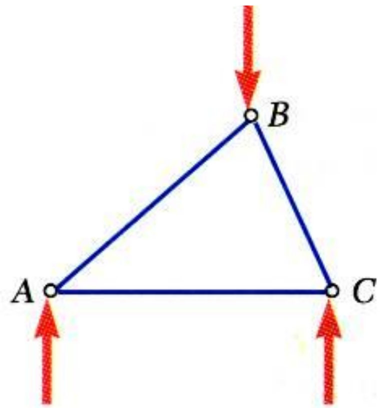
Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.

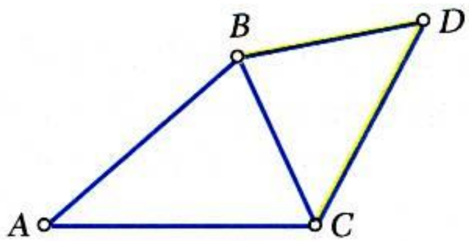
Definition of a Truss

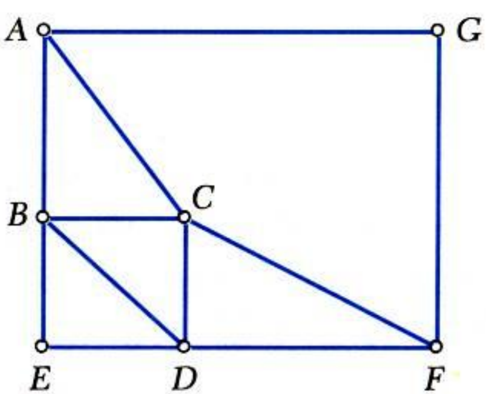


Simple Trusses







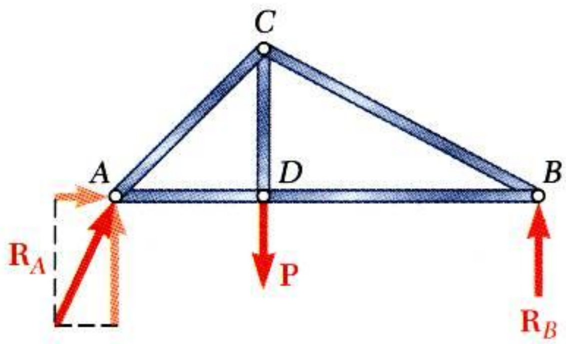


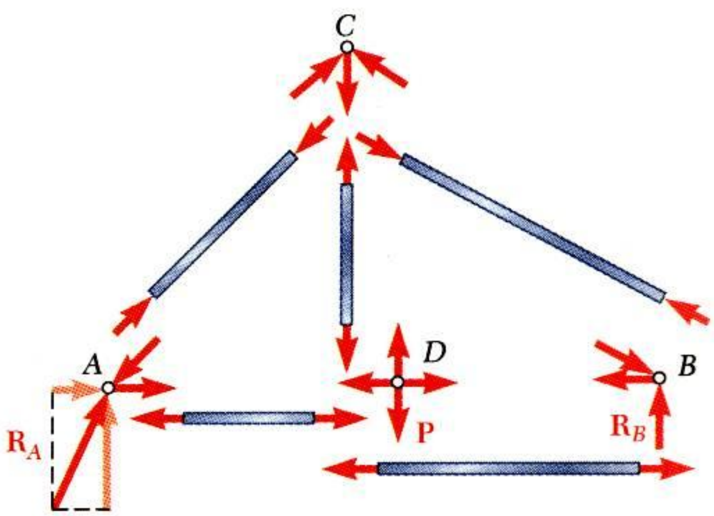
- A *rigid truss* will not collapse under the application of a load.

- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.

- In a simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.

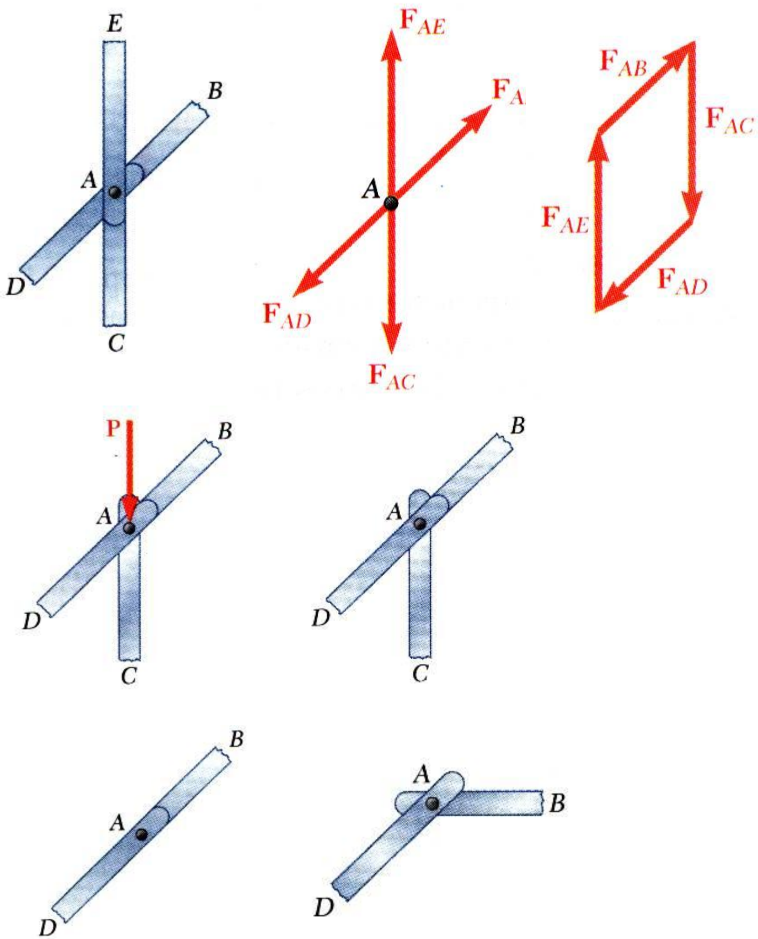
Analysis of Trusses by the Method of Joints



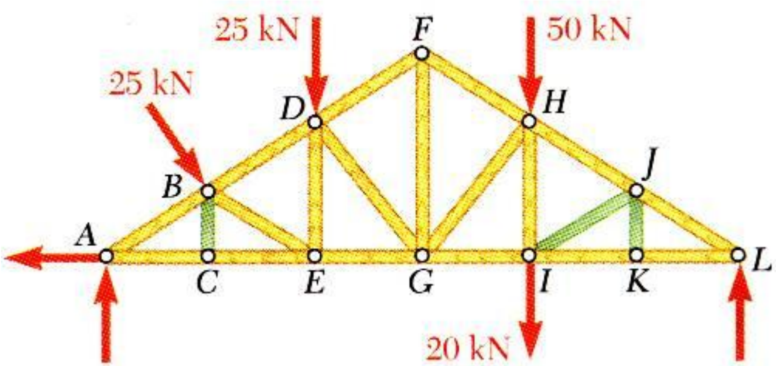


- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

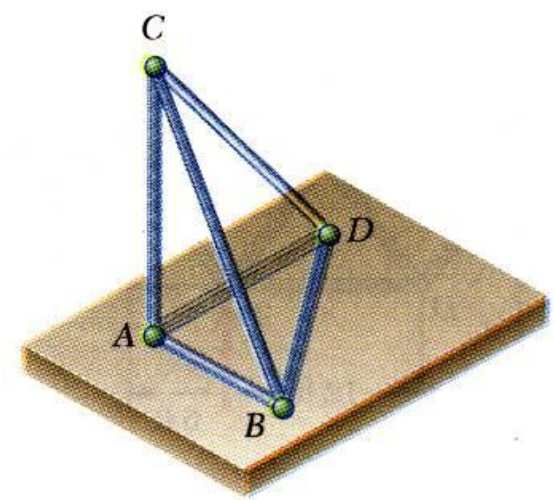
Joints Under Special Loading Conditions

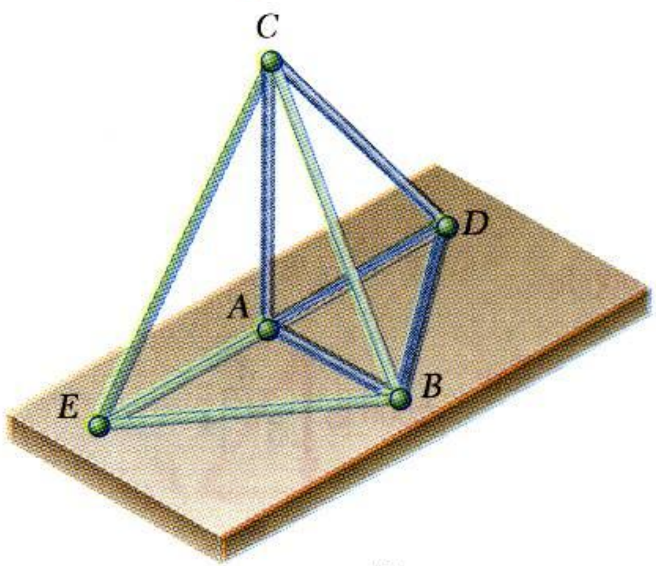


- Forces in opposite members intersecting in two straight lines at a joint are equal.
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.
- Recognition of joints under special loading conditions simplifies a truss analysis.



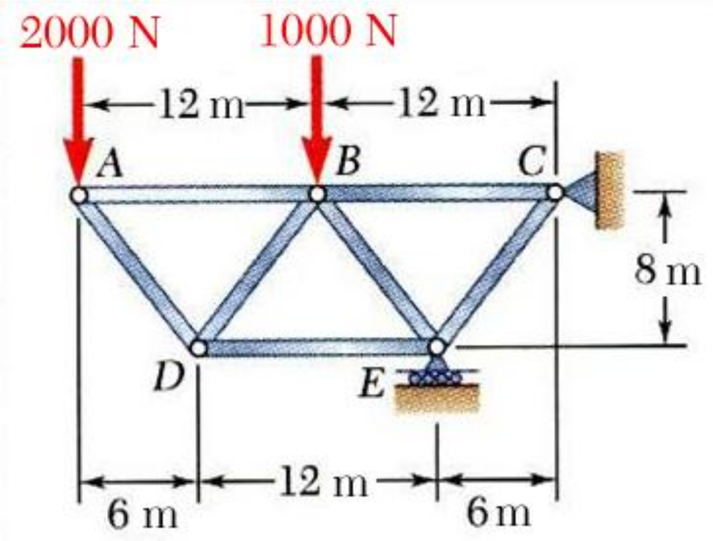
Space Trusses





- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss, $m = 3n - 6$ where m is the number of members and n is the number of joints.
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

Sample Problem 6.1

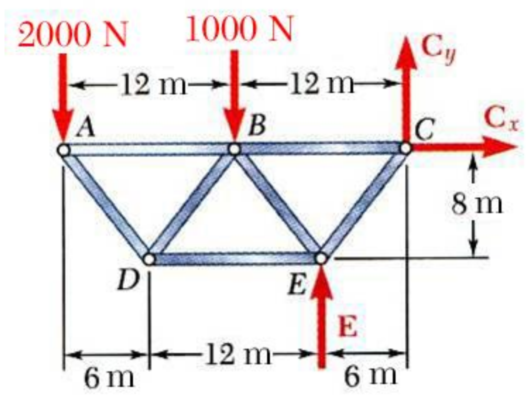


Using the method of joints, determine the force in each member of the truss.

<u>SOLUTION</u>:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints *D*, *B*, and *E* from joint equilibrium requirements.
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

Sample Problem 6.1



SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.

$$\sum M_C = 0$$

$$= (2000\ \text{N})(24\ \text{m}) + (1000\ \text{N})(12\ \text{m}) - E(6\ \text{m})$$

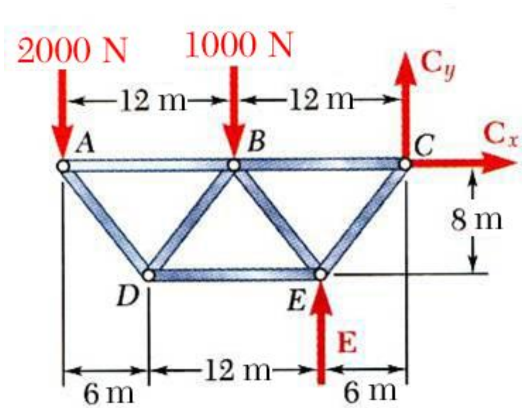
$$\boxed{E = 10{,}000\ \text{N} \uparrow}$$

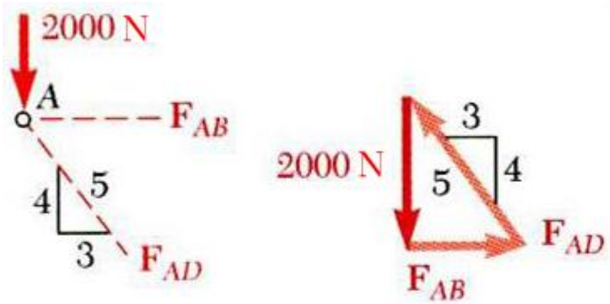
$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

$$\sum F_y = 0 = -2000\ \text{N} - 1000\ \text{N} + 10{,}000\ \text{N} + C_y$$

$$\boxed{C_y = 7000\ \text{N} \downarrow}$$

Sample Problem 6.1

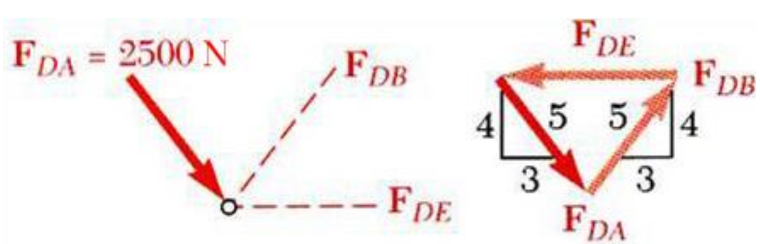




- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{2000\text{ N}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$F_{AB} = 1500\text{ N}\ \ T$$
$$F_{AD} = 2500\text{ N}\ \ C$$

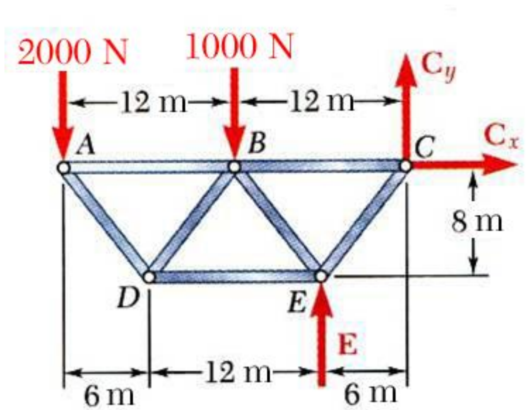


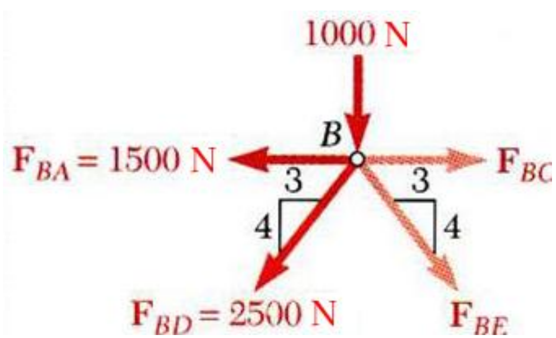
- There are now only two unknown member forces at joint D.

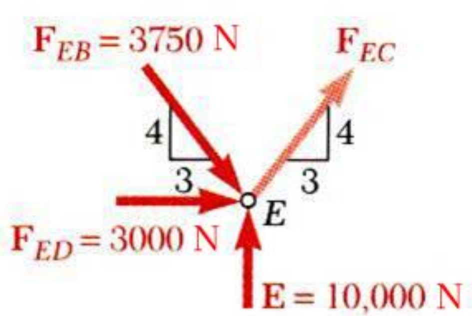
$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$F_{DB} = 2500\text{ N}\ \ T$$
$$F_{DE} = 3000\text{ N}\ \ C$$

Sample Problem 6.1







- There are now only two unknown member forces at joint B. Assume both are in tension.

$$\sum F_y = 0 = -1000 - \tfrac{4}{5}(2500) - \tfrac{4}{5}F_{BE}$$

$$F_{BE} = -3750\ \text{N} \qquad \boxed{F_{BE} = 3750\ \text{N}\ C}$$

$$\sum F_x = 0 = F_{BC} - 1500 - \tfrac{3}{5}(2500) - \tfrac{3}{5}(3750)$$

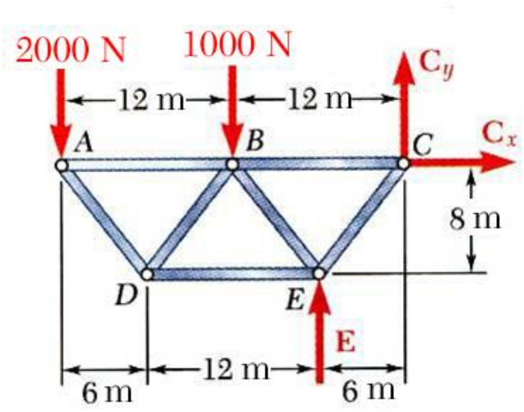
$$F_{BC} = +5250\ \text{N} \qquad \boxed{F_{BC} = 5250\ \text{N}\ T}$$

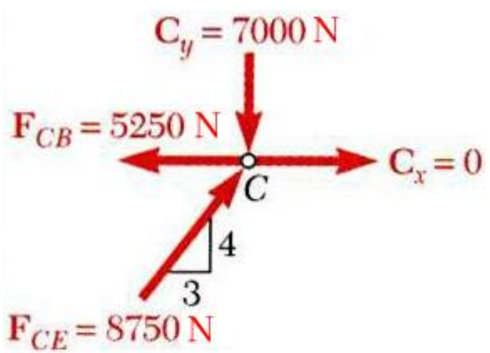
- There is one unknown member force at joint *E*. Assume the member is in tension.

$$\sum F_x = 0 = \tfrac{3}{5}F_{EC} + 3000 + \tfrac{3}{5}(3750)$$

$$F_{EC} = -8750\ \text{N} \qquad \boxed{F_{EC} = 8750\ \text{N}\ C}$$

Sample Problem 6.1



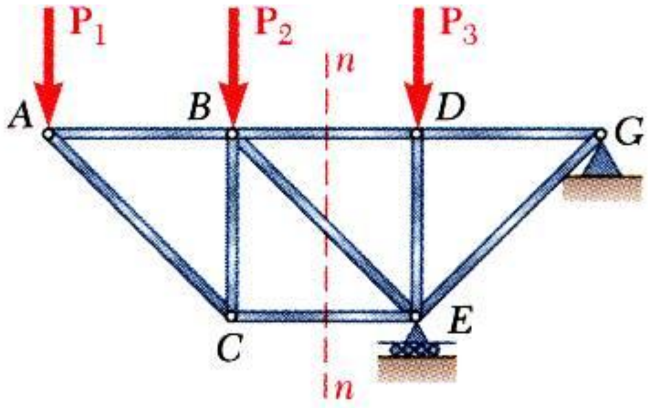


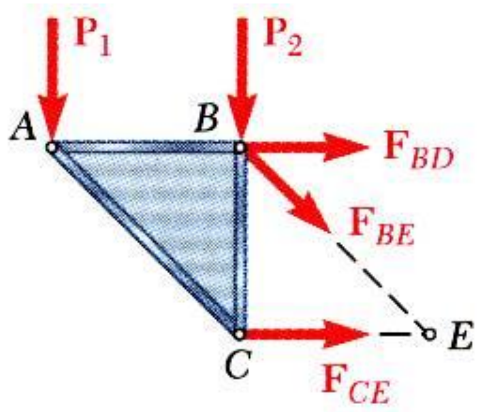
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

$$\sum F_x = -5250 + \tfrac{3}{5}(8750) = 0 \quad \text{(checks)}$$

$$\sum F_y = -7000 + \tfrac{4}{5}(8750) = 0 \quad \text{(checks)}$$

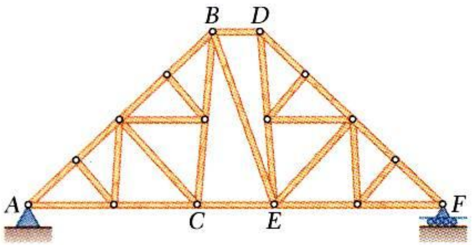
Analysis of Trusses by the Method of Sections

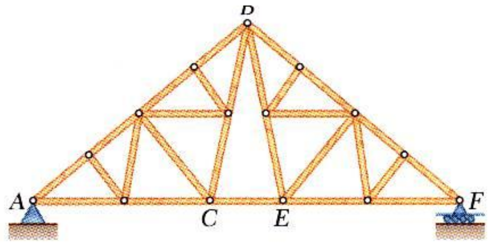


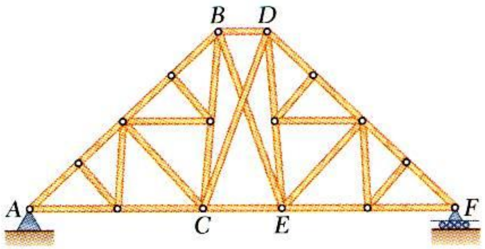


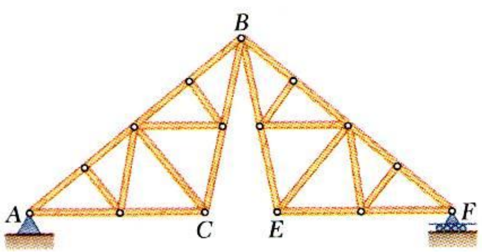
- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD*, *pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.

Trusses Made of Several Simple Trusses



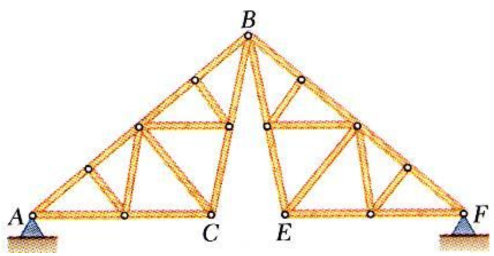






non-rigid

$$m < 2n - 3$$



rigid

$$m < 2n - 4$$

- *Compound trusses* are statically determinant, rigid, and completely constrained.

$$m = 2n - 3$$

- Truss contains a *redundant member* and is *statically indeterminate*.

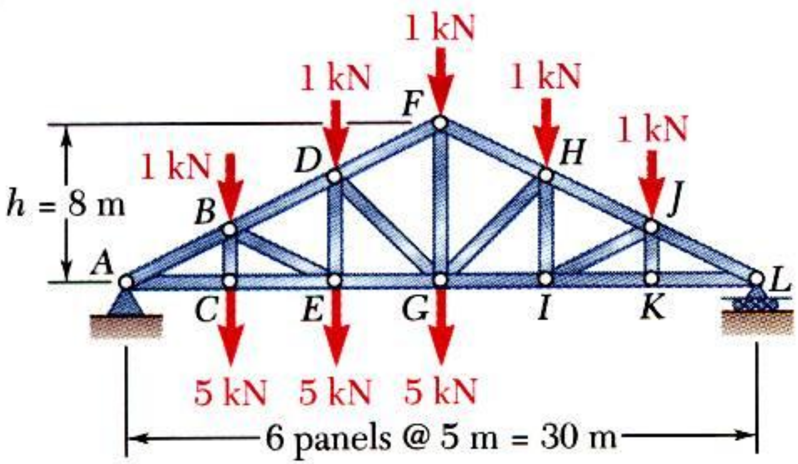
$$m > 2n - 3$$

- Additional reaction forces may be necessary for a rigid truss.

- Necessary but insufficient condition for a compound truss to be statically determinant, rigid, and completely constrained,

$$m + r = 2n$$

Sample Problem 6.3

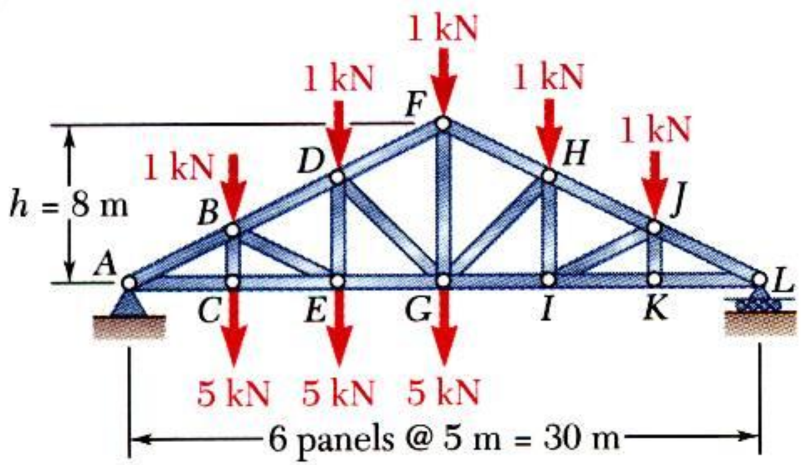


Determine the force in members *FH*, *GH*, and *GI*.

<u>SOLUTION</u>:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.
- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.

Sample Problem 6.3



SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.

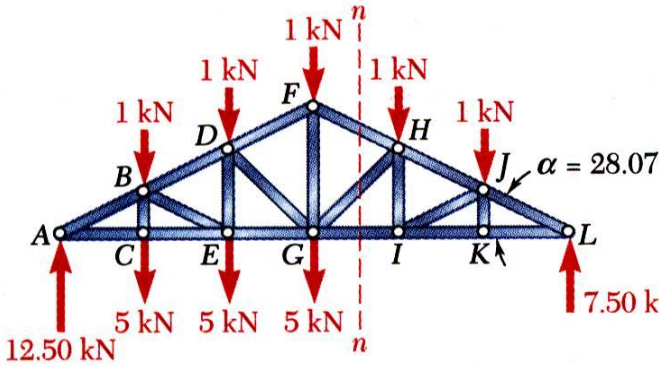
$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN})$$
$$-(20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L$$

$$L = 7.5\text{ kN} \uparrow$$

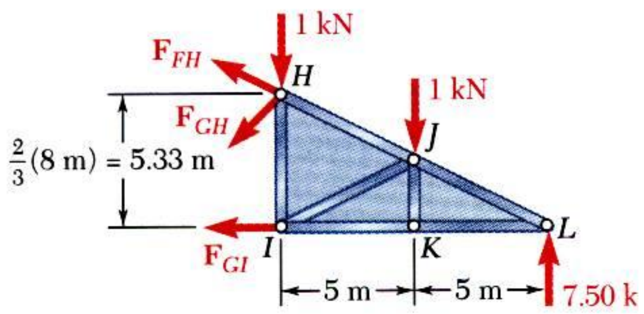
$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

Sample Problem 6.3



- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.



- Apply the conditions for static equilibrium to determine the desired member forces.

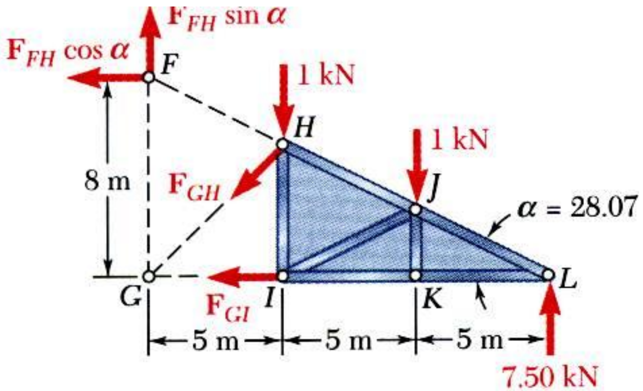
$$\sum M_H = 0$$

$$(7.50\,\text{kN})(10\,\text{m}) - (1\,\text{kN})(5\,\text{m}) - F_{GI}(5.33\,\text{m}) = 0$$

$$F_{GI} = +13.13\,\text{kN}$$

$$\boxed{F_{GI} = 13.13\,\text{kN} \quad T}$$

Sample Problem 6.3



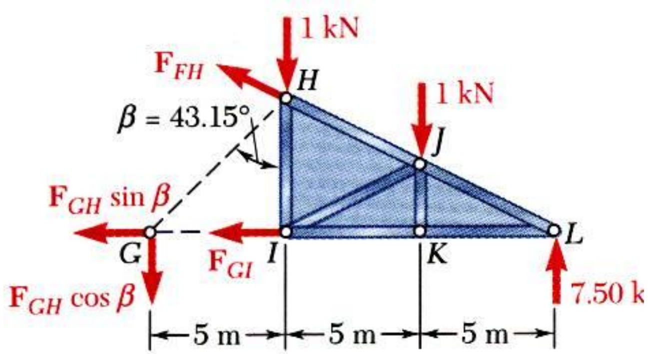
$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.82\text{ kN}$$

$$\boxed{F_{FH} = 13.82\text{ kN} \;\; C}$$



$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15°$$

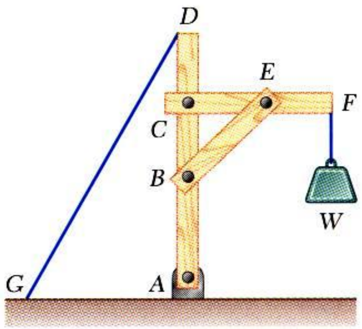
$$\sum M_L = 0$$

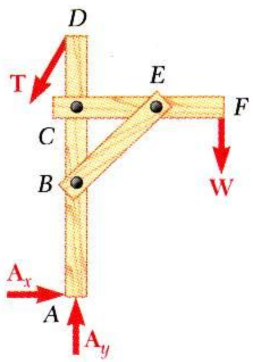
$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(10\text{ m}) = 0$$

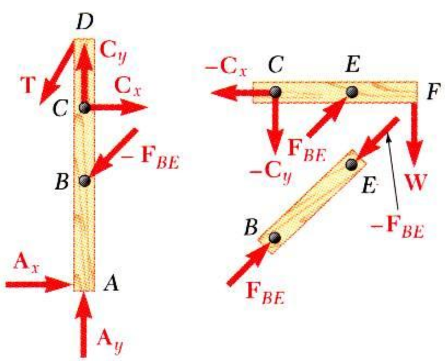
$$F_{GH} = -1.371\text{ kN}$$

$$\boxed{F_{GH} = 1.371\text{ kN} \;\; C}$$

Analysis of Frames

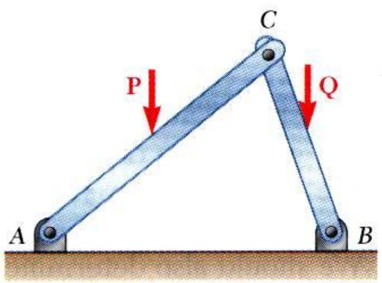


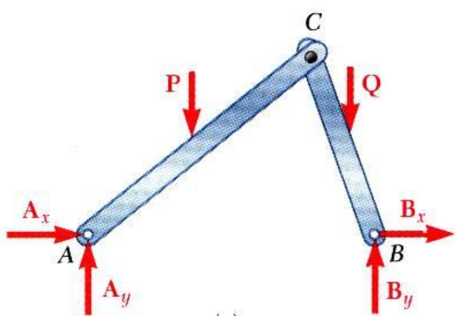


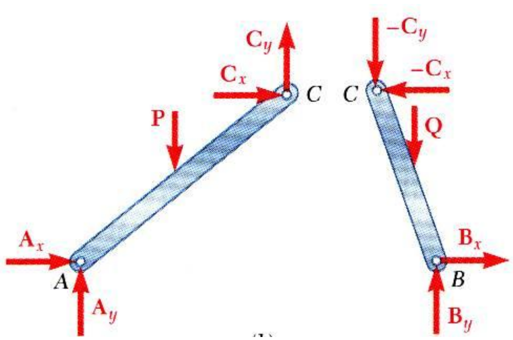


- *Frames* and *machines* are structures with at least one *multiforce* member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.
- Forces between connected components are equal, have the same line of action, and opposite sense.

Frames Which Cease To Be Rigid When Detached From Their Supports

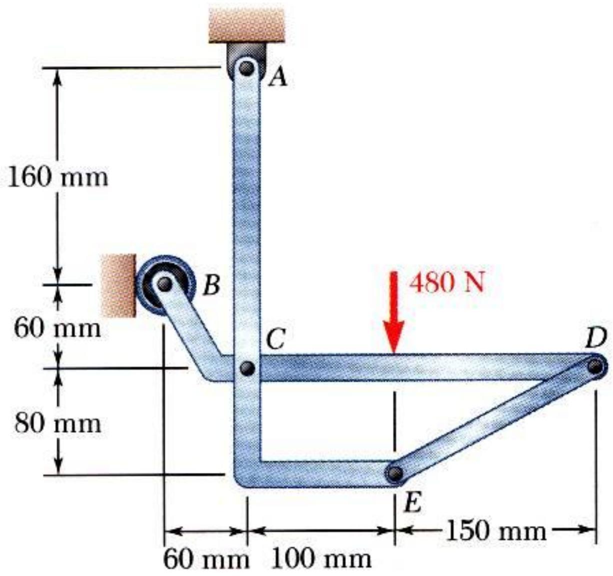






- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.
- A free-body diagram of the complete frame indicates four unknown force components which can not be determined from the three equilibrium conditions.
- The frame must be considered as two distinct, but related, rigid bodies.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams indicate 6 unknown force components.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations.

Sample Problem 6.4

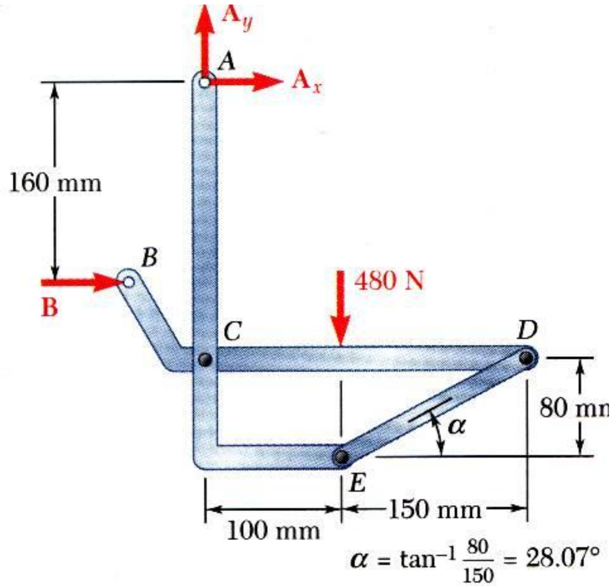


Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.
- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.
- With the force on the link *DE* known, the sum of forces in the *x* and *y* directions may be used to find the force components at *C*.
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

Sample Problem 6.4



SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480\text{ N} \qquad A_y = 480\text{ N} \uparrow$$

$$\sum M_A = 0 = -(480\text{ N})(100\text{ mm}) + B(160\text{ mm})$$

$$B = 300\text{ N} \rightarrow$$

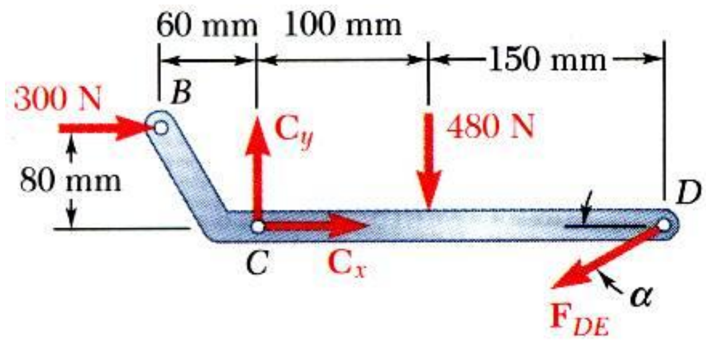
$$\sum F_x = 0 = B + A_x \qquad A_x = -300\text{ N} \leftarrow$$

Note:

$$\alpha = \tan^{-1}\frac{80}{150} = 28.07°$$

Sample Problem 6.4

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.



$$\sum M_C = 0 = (F_{DE}\sin\alpha)(250\text{ mm}) + (300\text{ N})(60\text{ mm}) + (480\text{ N})(100\text{ mm})$$

$$F_{DE} = -561\text{ N}$$

$$\boxed{F_{DE} = 561\text{ N } C}$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\text{ N}$$

$$0 = C_x - (-561\text{ N})\cos\alpha + 300\text{ N}$$

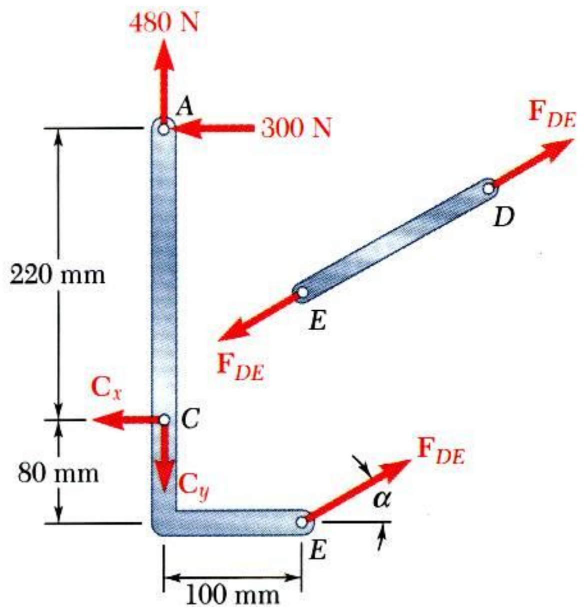
$$\boxed{C_x = -795\text{ N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\text{ N}$$

$$0 = C_y - (-561\text{ N})\sin\alpha - 480\text{ N}$$

$$\boxed{C_y = 216\text{ N}}$$

Sample Problem 6.4

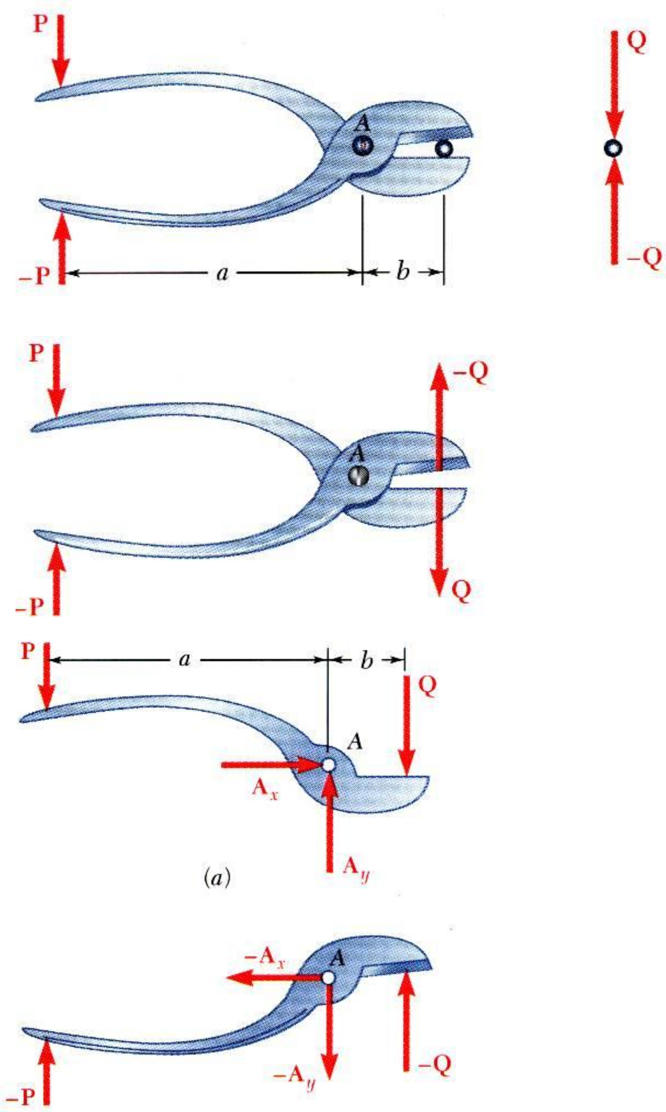


- With member *ACE* as a free-body, check the solution by summing moments about *A*.

$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

(checks)

Machines



- Machines are structures designed to transmit and modify forces. Their main purpose is to transform *input forces* into *output forces*.
- Given the magnitude of ***P***, determine the magnitude of ***Q***.
- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.
- The machine is a nonrigid structure. Use one of the components as a free-body.
- Taking moments about *A*,

$$\sum M_A = 0 = aP - bQ \qquad Q = \frac{a}{b} P$$